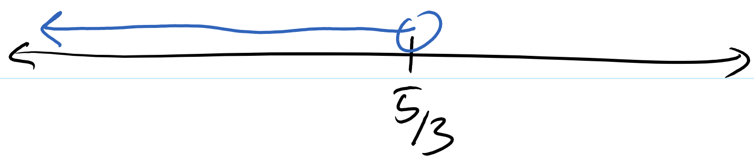
Inequalities: Section 3.6

- $3x - 5 < 0$

$$\begin{aligned} 3x - 5 &< 0 \\ +5 \quad & \quad +5 \\ \frac{3x}{3} &< \frac{5}{3} \\ x &< 5/3 \end{aligned}$$

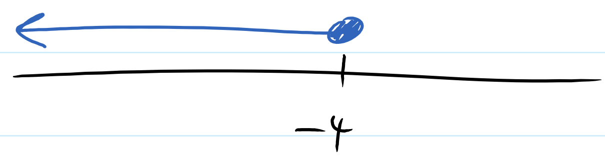


$(-\infty, 5/3)$

- $2 - x \geq 6$

$$\begin{aligned} 2 - x &\geq 6 \\ -2 \quad & \quad -2 \\ \frac{-x}{-1} &\geq \frac{4}{-1} \\ x &\leq -4 \end{aligned}$$

$$\begin{aligned} -x &\geq 4 \\ +x \quad & \quad +x \\ 0 &\geq 4 + x \\ -4 \quad & \quad -4 \\ -4 &\geq x \end{aligned}$$

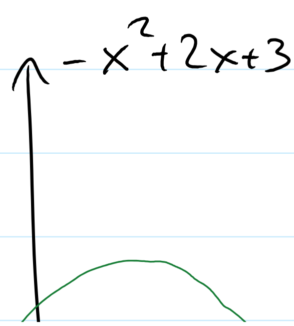


$(-\infty, -4]$

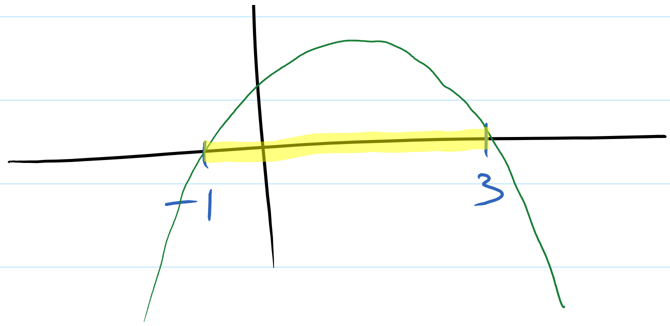
- $(-x+3)(x+1) > 0$

$(3-x)(x+1) > 0$

$-x^2 + 2x + 3 > 0$



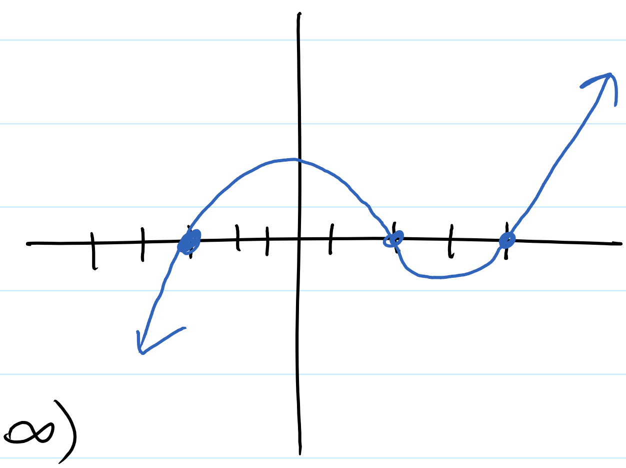
$$-x^2+2x+3 > 0$$



$$\boxed{(-1, 3)}$$

- Given a graph of $f(x)$, solve:

$$f(x) \le 0$$



sign chart

	$(-\infty, -3)$	$(-3, 2)$	$(2, 4)$	$(4, \infty)$
$f(x)$	$-$	$+$	$-$	$+$

solution: $\boxed{(-\infty, -3] \cup [2, 4]}$

- Make a sign chart for $f(x) = (3-x)(x+1)$ and use it to solve:

$$f(x) \ge 0.$$

sign chart: 1) $f(x)=0$

2) determine if the endpts are included (from the inequality)

3) find the sign of $f(x)$ on each interval.

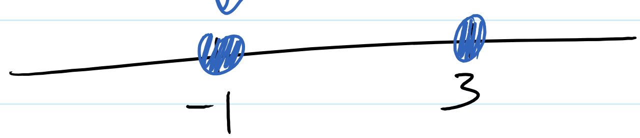
1) $(3-x)(x+1)=0$

$3-x=0$ $x+1=0$

$3=x$ $x=-1$

$(3-x)(x+1)\geq 0$

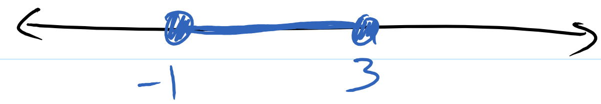
x=-1 satisfies the inequality



	$(-\infty,-1]$	$[-1,3]$	$[3,\infty)$
	-2	0	4
sign of $f(x)$	$(3-(-2))(-2+1)$ $5\cdot(-1)$ $-$	$(3-0)(0+1)$ $(3)(1)$ $+$	$(3-4)(4+1)$ $(-1)(5)$ $-$

solution: $\boxed{[-1,3]} \Rightarrow -1\leq x\leq 3$

$3\geq x\geq -1$

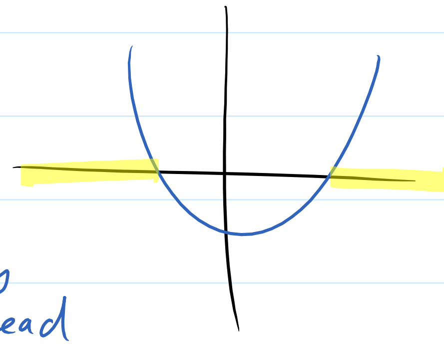


Solve:

$$2x^2 + x > 15$$
$$-15 \quad -15$$

get zero on one side

$$2x^2 + x - 15 > 0$$



solve equality instead

$$2x^2 + x - 15 = 0$$
$$2x^2 + 6x - 5x - 15 = 0$$
$$2x(x+3) - 5(x+3) = 0$$
$$(x+3)(2x-5) = 0$$

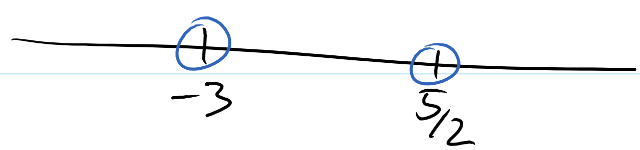
$A \cdot C = -30$

$A + C = 1$

$x + 3 = 0$ $\qquad$ $2x - 5 = 0$

$x = -3$ $\qquad$ $x = \frac{5}{2}$

$(x+3)(2x-5) > 0$



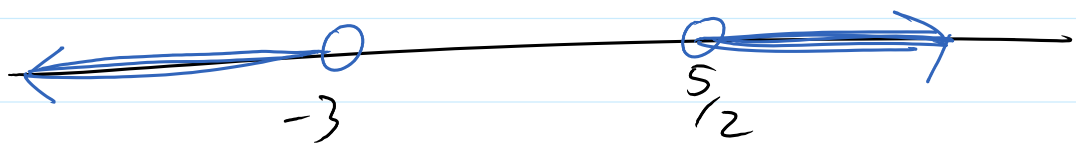
	$(-\infty, -3)$	$(-3, \frac{5}{2})$	$(\frac{5}{2}, \infty)$
	-4	0	10
$x+3$	$-$	$+$	$+$
$2x-5$	$-$	$-$	$+$
$f(x)$	$+$	$-$	$+$

All factors of the function

$2x-5$	$-$		$+$
$f(x)$	$+$	$-$	$+$

the function

$$\boxed{(-\infty, -3) \cup (5/2, \infty)}$$



Solve: $4x^2 \le 1 - 2x$

$$4x^2 - 1 + 2x \le 0$$
$$4x^2 + 2x - 1 \le 0$$

$$4x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4\cdot 4\cdot(-1)}}{2\cdot 4}$$

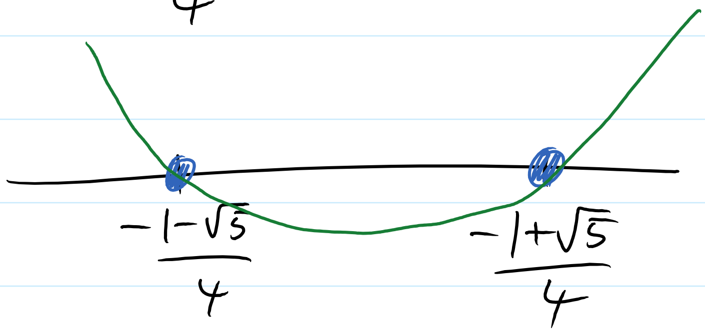
$A \cdot C = -4$
$A + C = 2$

$$= \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm \sqrt{20}}{8} = \frac{-2 \pm \sqrt{4}\cdot\sqrt{5}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{} = \frac{\cancel{2}(-1 \pm \sqrt{5})}{} = \frac{-1 \pm \sqrt{5}}{}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8} = \frac{\cancel{2}(-1 \pm \sqrt{5})}{\cancel{8}_{4}} = \frac{-1 \pm \sqrt{5}}{4}$$

$4x^2 + 2x - 1 \le 0$



Since the function graph is a parabola open up, with zeros at $\frac{-1 \pm \sqrt{5}}{4}$

The graph is below the x-axis between $\frac{-1-\sqrt{5}}{4}$ and $\frac{-1+\sqrt{5}}{4}$

Solution $\boxed{\left[\frac{-1-\sqrt{5}}{4}, \frac{-1+\sqrt{5}}{4}\right]}$

Solve: $x^3 + x^2 \le 4x + 4$

$$x^3 + x^2 - 4x - 4 \le 0$$

$$\underbrace{x^3 + x^2} - \underbrace{4x - 4} = 0$$

$$x^2(x+1) - 4(x+1) = 0$$

$$(x^2 - 4)(x+1) = 0$$

$$(x^2-4)(x+1)=0$$

$$(x-2)(x+2)(x+1)=0$$

$\underline{x=2}$ $\quad$ $\underline{x=-2}$ $\quad$ $\underline{x=-1}$

	$(-\infty,-2]$	$[-2,-1]$	$[-1,2]$	$[2,\infty)$
pt	-3	-1.5	0	3
$x-2$	$-$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$	$+$
$x+1$	$-$	$-$	$+$	$+$
$f(x)$	$-$	$+$	$-$	$+$

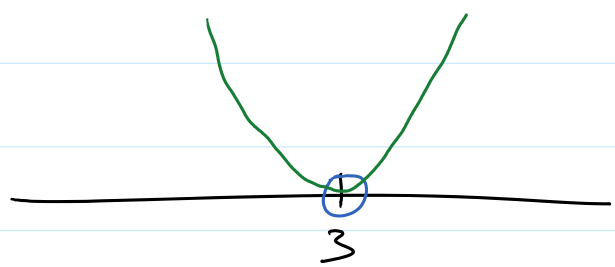
$$\boxed{(-\infty,-2]\cup[-1,2]}$$

Solve: $x^2-6x+9<0$

$$x^2-6x+9=0$$

$$(x-3)(x-3)=0$$

$$(x-3)^2=0$$



$$x-3=0$$

$x=3$

	$(-\infty, 3)$	$(3, \infty)$
pt.	0	4
$(x-3)^2$	+	+

$\boxed{\text{no solution}}$ $\emptyset$