11/02

Thursday, November 2, 2017 4:59 PM

Ex: Solve: $x^4 \geq 1$

$-1 \quad -1$

$x^4 - 1 \geq 0$

$x^4 - 1 = 0$

$(x^2)^2 - 1^2 = 0$

$(x^2-1)(x^2+1) = 0$

$(x-1)(x+1)(x^2+1) = 0$

$x-1=0 \qquad x+1=0 \qquad x^2+1=0$

$x = 1 \qquad x = -1 \qquad x^2 = -1$

none

	$(-\infty, -1]$	$[-1, 1]$	$[1, \infty)$
pt	-2	0	2
$x-1$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$
x^2+1	$+$	$+$	$+$
$f(x)$	$+$	$-$	$+$

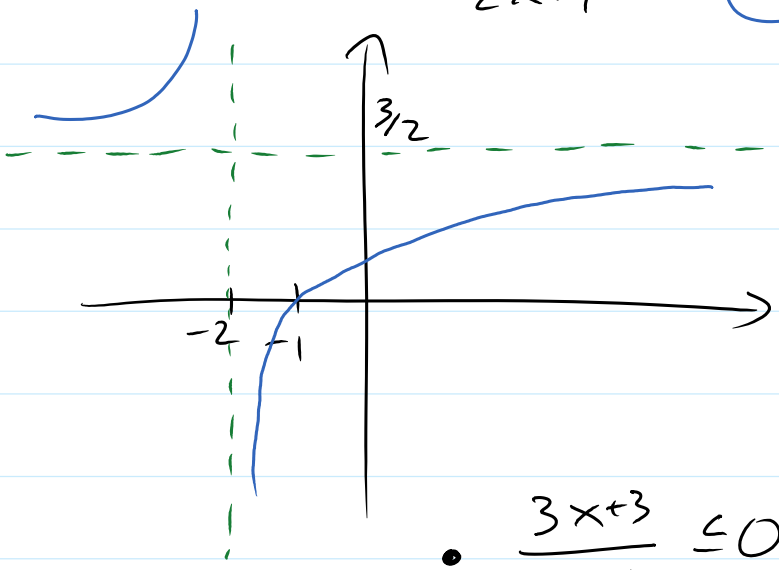
$(-\infty, -1] \cup [1, \infty)$

Section 3.6 - Rational inequalities

Ex: Solve: $\frac{3x+3}{2x+4} > 0$

→ The graph of $f(x) = \frac{3x+3}{2x+4}$ is above the x-axis:

$$(-\infty, -2) \cup (-1, \infty)$$



- $\frac{3x+3}{2x+4} \leq 0$

$$(-2, -1]$$

-2 is not included since $x=-2$ is not in the domain

Ex: Solve:

$$\frac{x+1}{x+3} \geq 2$$

$$\frac{x+1}{x+3} - \frac{2}{1} \geq 0$$

solve ... $=0$

$$\frac{x+1}{x+3} - \frac{2}{1} = 0$$

$$\frac{x+1}{x+3} - \frac{2}{1} \cdot \frac{x+3}{x+3} = 0$$

$$\frac{x+1}{x+3} - \frac{2}{1} \cdot \frac{x+3}{x+3} = 0$$

$$\frac{x+1}{x+3} - \frac{2(x+3)}{x+3} = 0$$

$$\frac{x+1-2(x+3)}{x+3} = 0$$

$$\frac{x+1-2x-6}{x+3} = 0$$

$$\frac{-x-5}{x+3} = 0$$

den = 0

num = 0

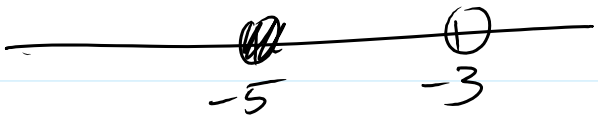
$-x-5=0$

$-x=5$

$x=-5$

$x+3=0$

$x=-3$



	$(-\infty,-5]$	$[-5,-3)$	$(-3,\infty)$
pt	-6	-4	0
$-x-5$	$+$	$-$	$-$
$x+3$	$-$	$-$	$+$
$f(x)$	$-$	$+$	$-$

$f(x) \geq 0$

$$[-5,-3)$$

$$\boxed{-5 \le x < -3}$$

Ex: Solve: $\dfrac{3x+5}{6-2x} \ge 0$

$3x+5=0$

$3x=-5$

$x=-5/3$

$6-2x=0$

$-2x=-6$

$x=3$

	$(-\infty, -5/3]$	$[-5/3, 3)$	$(3, \infty)$
pt	-3	0	4
$3x+5$	$-$	$+$	$+$
$6-2x$	$+$	$+$	$-$
$f(x)$	$-$	$+$	$-$

$f(x) \ge 0$

$$\boxed{[-5/3, 3)}$$

$$\boxed{-\frac{5}{3} \le x < 3}$$

$$\frac{(x+3)^2(x-2)}{x+1} \le 0$$

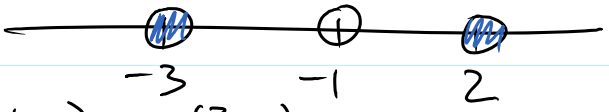
$$\frac{(x+3)^2(x-2)}{x+1} \leq 0$$

$(x+3)^2(x-2)=0$ $\qquad$ $x+1=0$

$x=-1$

$x+3=0$ $\qquad$ $x-2=0$

$x=-3$ $\qquad$ $x=2$



"<" →	$(-\infty,-3)$	$(-3,-1)$	$(-1,2)$	$(2,\infty)$
"≤" →	$(-\infty,-3]$	$[-3,-1)$	$(-1,2]$	$[2,\infty)$
pt	-4	-2	0	3
$(x+3)^2$	+	+	+	+
$x-2$	−	−	−	+
$x+1$	−	−	+	+
$f(x)$	+	+	−	+

"<" → $\boxed{(-1,2)}$

"≤" → $\boxed{(-1,2]}$

Ex: Find the domain of

$$f(x)=\sqrt{\frac{x}{2x-1}-1}$$

The domain of $\sqrt{x}$ is $[0,\infty)$

$$\frac{x}{2x-1}-1 \geq 0$$

$$\frac{x}{2x-1} - 1 \geq 0$$

$$\frac{x}{2x-1} - \frac{1}{1} \geq 0$$

$\frac{A}{B} - \frac{C}{D} = \frac{AD-CB}{BD}$

$$\frac{x\cdot 1 - 1\cdot(2x-1)}{(2x-1)\cdot 1} \geq 0$$

$$\frac{x - 2x + 1}{2x-1} \geq 0$$

$$\frac{-x+1}{2x-1} \geq 0$$

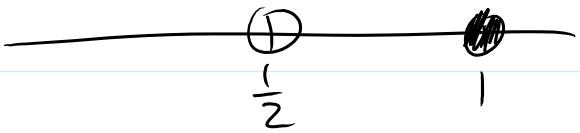
$-x+1=0$

$-x=-1$

$x=1$

$2x-1=0$

$x=1/2$



	$(-\infty, \frac{1}{2})$	$(\frac{1}{2}, 1]$	$[1, \infty)$
pt	0	0.7	2
$-x+1$	+	+	−
$2x-1$	−	+	+
$f(x)$	−	(+)	−

$f(x) \geq 0$

The domain is $(\frac{1}{2}, 1]$

Solve: $|x^2+6x+1| > 8$ → solve equality first

$|x^2+6x+1| = 8$

$x^2+6x+1 = 8$

$x^2+6x-7=0$

$(x+7)(x-1)=0$

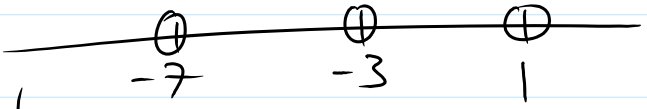
$x=-7$, $x=1$

$x^2+6x+1=-8$

$x^2+6x+9=0$

$(x+3)^2=0$

$x=-3$



	$(-\infty,-7)$	$(-7,-3)$	$(-3,1)$	$(1,\infty)$
pt	-8	-4	0	2
$\|x^2+6x+1\|>8$	$\|64-48+1\|>8$	$\|16-24+1\|>8$	$\|1\|>8$	$\|4+12+1\|>8$
	$\|17\|>8$	$\|-7\|>8$	$1>8$	$\|17\|>8$
	✓	$7>8$	×	$17>8$
		×		✓

$$(-\infty,-7) \cup (1,\infty)$$

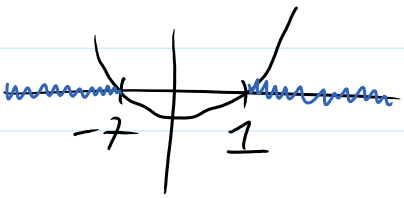
Another way of approaching this problem:

$|x^2+6x+1| > 8$

$x^2+6x+1 > 8$ or $x^2+6x+1 < -8$

$x^2+6x-7 > 0$ $\quad$ $x^2+6x+9 < 0$

$(x-1)(x+7) > 0$ $\quad$ $(x+3)^2 < 0$



$(-\infty, -7) \cup (1, \infty)$

$x = -3$

no solution

put the solutions together

$\boxed{(-\infty, -7) \cup (1, \infty)}$