11/07

Tuesday, November 7, 2017 3:25 PM

- Online office hour on Sunday 11/12 at 6:30PM (must end before 8PM)
- Offline HW due Tuesday 11/14.

Section 3.5

Find the domain:

$$g(x) = \frac{x}{x^2-9}$$

$$x^2-9 \neq 0$$
$$(x-3)(x+3) \neq 0$$
$$x \neq 3,\ x \neq -3$$

Dom: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

$\{x \mid x \neq 3 \text{ and } x \neq -3\}$

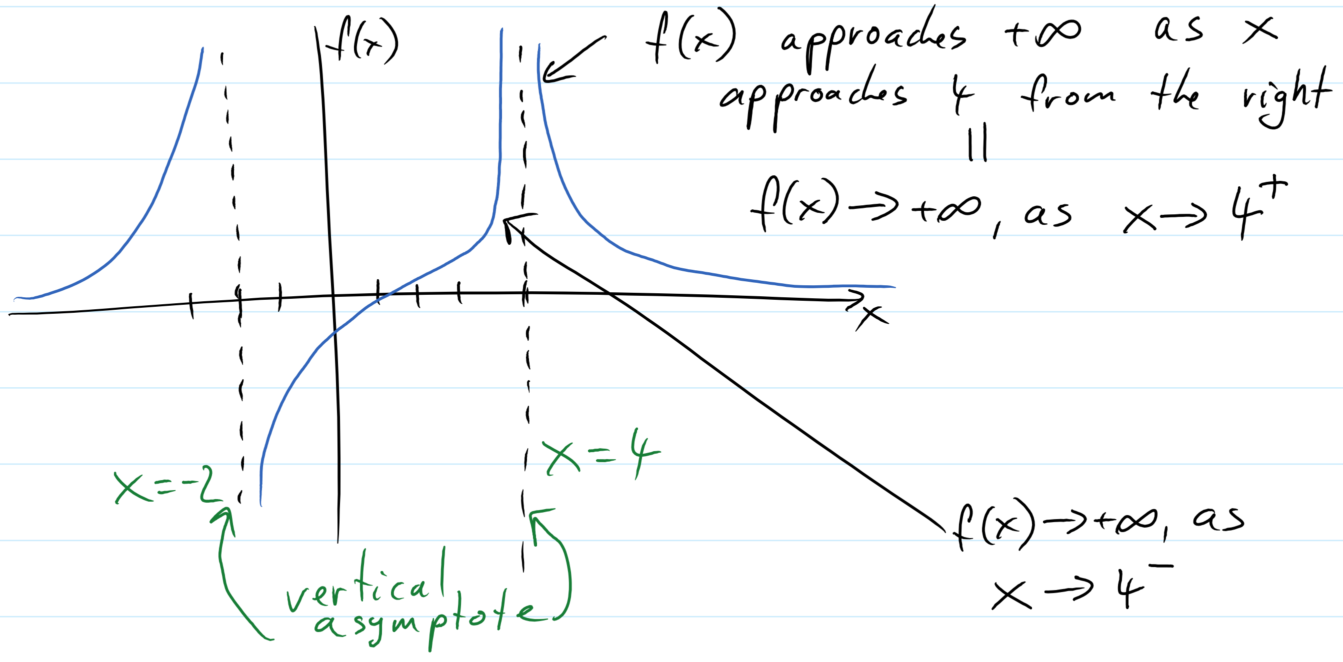
$$h(x) = \frac{x+3}{x^2+9}$$

$$x^2+9 \neq 0$$
$$x^2 \neq -9$$
$$x \neq \pm\sqrt{-9}$$

not real

Domain: $(-\infty, \infty)$

$\{x\} \Leftrightarrow$ all real numbers



- As $x \to -2^-$, $f(x) \to +\infty$
- As $x \to -2^+$, $f(x) \to -\infty$

Def: The line $x = a$ is a vertical asymptote of the graph of a function f if $f(x)$ increases or decreases without bound ($f(x) \to +\infty$ or $f(x) \to -\infty$) as x approaches a.

Note: A graph of a function that has a vertical asymptote $x = a$, never crosses this asymptote.

Locating vertical asymptotes

If $f(x)=\frac{p(x)}{q(x)}$ is a rational function in which $p(x)$ and $q(x)$ have no common factors and a is a zero of $q(x)$, then $x=a$ is a vertical asymptote.

$\frac{p(x)}{q(x)}$ is simplified!

Ex: find vert. asymptotes:

1) simplify
2) denom. $=0$

- $f(x)=\frac{x}{x^2-9}$ $\qquad \frac{x}{x^2-9}=\frac{x}{(x-3)(x+3)}$

$$(x-3)(x+3)=0$$

$x-3=0$ $\qquad$ $x+3=0$

$\underline{x=3}$ $\qquad$ $\underline{x=-3}$

$$\boxed{x=3,-3}$$

- $g(x)=\frac{x-1}{x^2-1}=\frac{x-1}{(x-1)(x+1)}=\frac{1}{x+1}$

$$x+1=0$$

$$\boxed{x=-1}$$

Domain: $\{x \mid x\neq 1 \text{ and } x\neq -1\}$

The graph has a hole at $x=1$

Domain: $\{x \mid x \neq 1 \text{ and } x \neq -1\}$

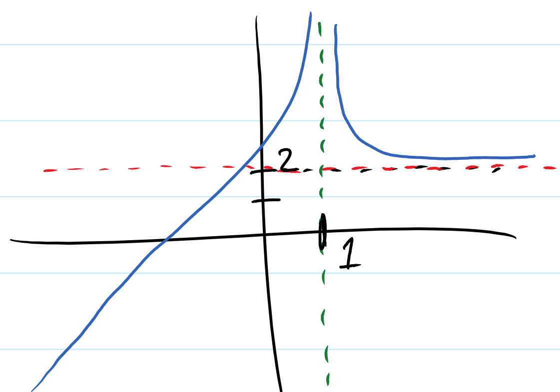
The graph has a hole at $x = 1$.

Horizontal asymptotes

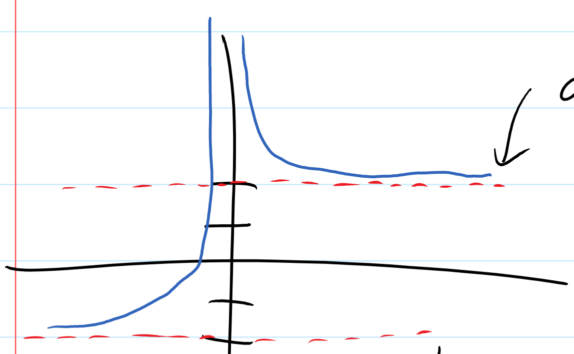
Def: The line $y = b$ is a horizontal asymptote of the graph of $f(x)$ if $f(x) \to b$ as $x \to +\infty$ or $x \to -\infty$.

Find hor. asymp:

$\boxed{y=0}$



$\boxed{y=2}$



as $x \to +\infty$, $f(x) \to 2$

vert. asy: $\boxed{x=0}$

hor. asymptotes: $\boxed{y=2 \text{ and } y=-2}$

locating horizontal asymptotes

let f be the rational function:

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0}$$

(degree of num: n
degree of denom: m)

1) If $m > n$ then the line $y=0$ is a horizontal asymptote.

2) If $m < n$ then the function has no horizontal asymptote

3) If $m=n$ the the line $y = \frac{a_n}{b_n}$ is a horizontal asymptote.

Ex: Find hor. asymp:

- $\frac{4x}{2x^2-1} \longrightarrow \boxed{y=0}$

- $\frac{4x^2}{2x^2 \dots} \longrightarrow y = \frac{4}{2} \quad \boxed{y=2}$

- $\dfrac{4x^2}{2x^2-1} \longrightarrow \quad y = \dfrac{4}{2} \quad \boxed{y=2}$

- $\dfrac{4x^3}{2x^2-1} \longrightarrow$ $\boxed{\text{no hor. asymptote}}$