11/09

Thursday, November 9, 2017 4:56 PM

- Online office hour on Sunday 11/12 at 6:30 PM (must end before 8PM)
- Offline HW due Tuesday 11/14.

$$f(x) = \frac{x-3}{x^2-9}$$

1) Domain: $x^2 - 9 \neq 0$

$x^2 \neq 9$

$x \neq \pm\sqrt{9}$

$x \neq \pm 3$

$\{x \mid x \neq 3 \text{ and } x \neq -3\}$

2) vertic. asympt: $\frac{x-3}{x^2-9} = \frac{\cancel{x-3}}{\cancel{(x-3)}(x+3)} = \frac{1}{x+3}$

there is a hole at $x = 3$

$x + 3 = 0$

$\boxed{x = -3}$

$$f(x) = \frac{(x-3)}{(x-3)(x^2-9)}$$

Dom: $x - 3 \neq 0$ $\quad x^2 - 9 \neq 0$

$x \neq 3$ $\quad x \neq \pm 3$

$\{x \mid x \neq \pm 3\}$

vert. asymp: $\frac{x-3}{(x-3)(x^2-9)} = \frac{1}{x^2-9}$

$x^2 - 9 = 0$

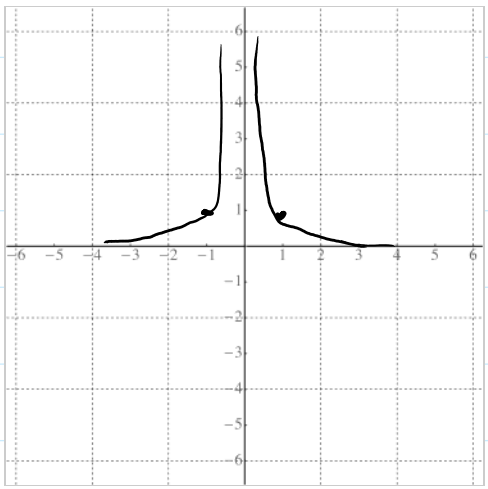
$x = \pm 3$

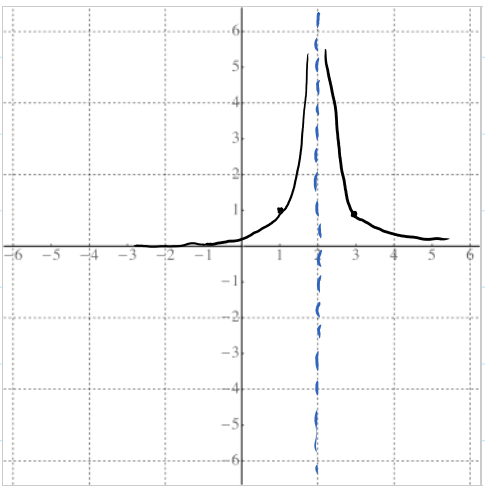
vert. asymp: $x=3,\ x=-3$

hor. asymp: $1 < 3 \Rightarrow y = 0$

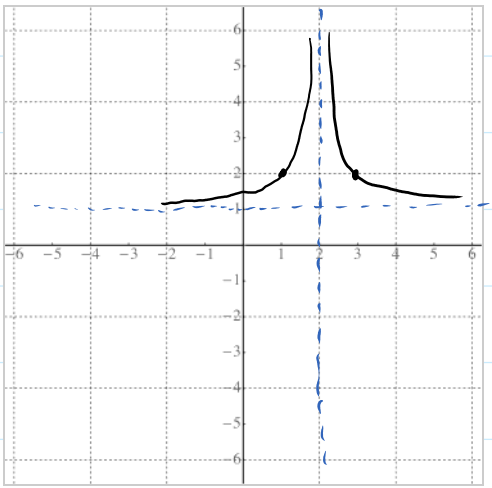
Ex: Graph $\frac{1}{(x-2)^2} + 1$ from $\frac{1}{x^2}$ using transformations.

$\frac{1}{x^2} \longrightarrow \frac{1}{(x-2)^2}$ hor. shift right by 2





$\frac{1}{(x-2)^2} + 1$ vert. shift up by 1



hor. as: $y=1$
vert. as: $x=2$

Graphing a rational function

$f(x)=\frac{p(x)}{q(x)}$

Find:

1) Domain
2) y-intercept (set $x=0$)
3) x-intercept (set $p(x)=0$)
4) vertical asymptotes (simplify & $q(x)=0$)
5) horizontal asymptotes
6) symmetry (odd: $f(-x)=-f(x)$; even: $f(-x)=f(x)$)
7) sign chart
8) plot the function.

Plot: $f(x)=\frac{2x-1}{x-1}$

Domain: $x-1\neq 0$, $x\neq 1$ $\quad \boxed{\{x \mid x\neq 1\}}$

y-int: $f(0)=\frac{0-1}{0-1}=1$ $\quad \boxed{(0,1)}$

y-int: $f(0) = \frac{-1}{0-1} = 1$ $\boxed{(0,1)}$

x-int: $2x-1=0$

$2x=1$

$x=\frac{1}{2}$ $\boxed{(\frac{1}{2}, 0)}$

vert. asy: $x-1=0$

$\boxed{x=1}$

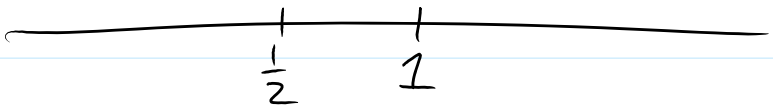
hor. asy: $\frac{2x-1}{x-1}$ degrees are the same (1)

$y=\frac{2}{1} \rightarrow \boxed{y=2}$

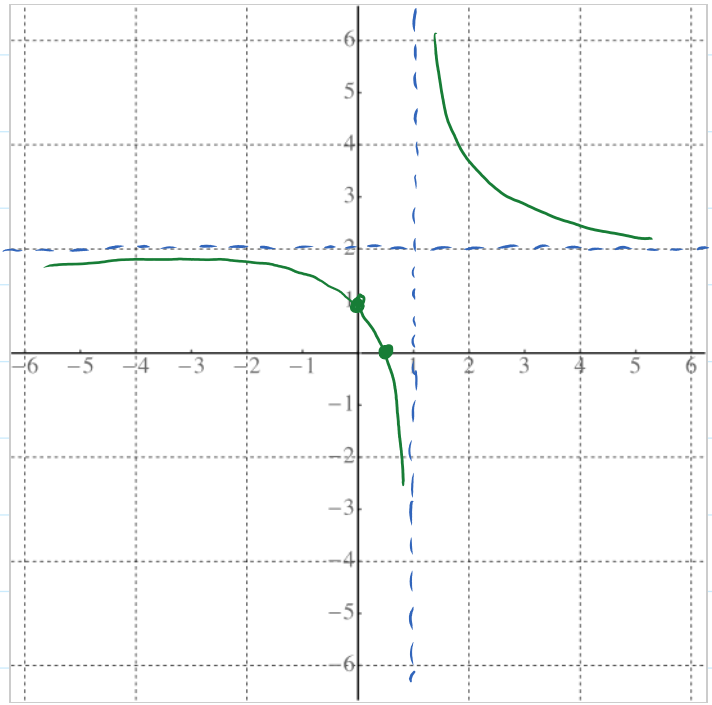
sym: $f(-x) = \frac{-2x-1}{-x-1} = \frac{-(2x+1)}{-(x+1)} = \frac{2x+1}{x+1}$

$\boxed{\text{neither}}$ $f(x)$ is not symmetric

sign chart:



	$(-\infty, \frac{1}{2})$	$(\frac{1}{2}, 1)$	$(1, \infty)$
	0	0.7	2
$2x-1$	$-$	$+$	$+$
$x-1$	$-$	$-$	$+$
$f(x)$	$+$	$-$	$+$



Graph: $\frac{3x^2}{x^2-4}$

Dom: $x^2 - 4 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

$\{x \mid x \neq \pm 2\}$

y-int: $f(0) = \frac{0}{0-4} = 0$ $\quad (0,0)$

x-int: $3x^2 = 0$ $\quad x = 0$ $\quad \boxed{(0,0)}$

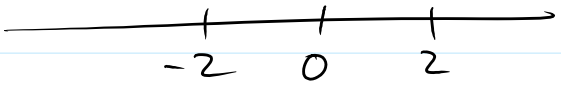
hor. asy: $y = \frac{3}{1} \rightarrow \boxed{y = 3}$

vert. asy: $x^2 - 4 = 0$ $\quad x^2 = 4$ $\quad \boxed{x = \pm 2}$

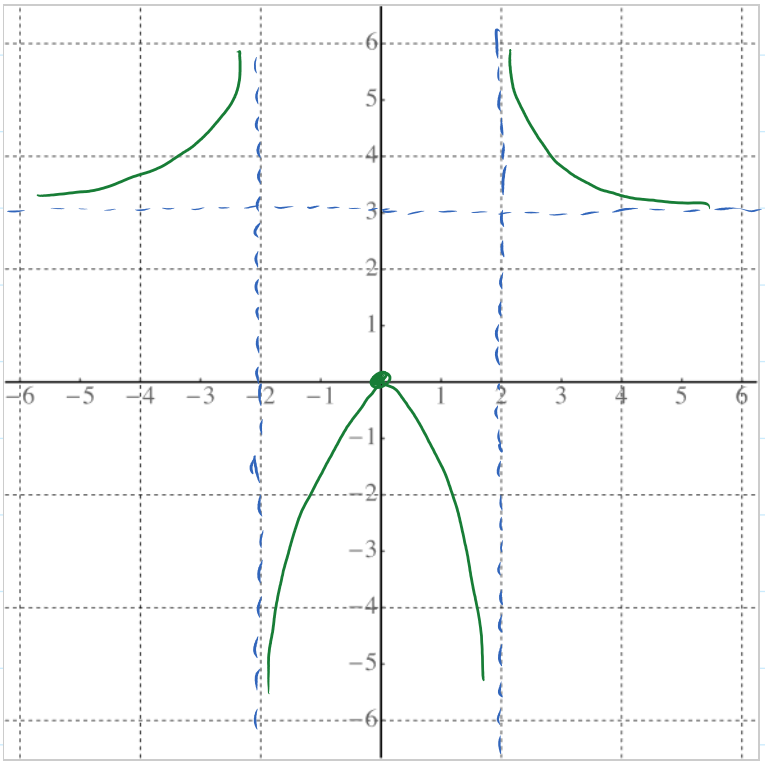
sym: $f(-x) = \frac{3(-x)^2}{(-x)^2 - 4} = \frac{3x^2}{x^2 - 4} = f(x)$

f is $\boxed{\text{even}}$

sign chart:



	$(-\infty, -2)$	$(-2, 0)$	$(0, 2)$	$(2, \infty)$
pt	-3	-1	1	3
$3x^2$	$+$	$+$	$+$	$+$
$x^2 - 4$	$+$	$-$	$-$	$+$
	$+$	$-$	$-$	$+$



Graph: $f(x) = \dfrac{-x^3+3x^2}{x^2-9}$

Dom: $x^2-9 \neq 0$

$x^2 \neq 9$

$x \neq \pm 3$

$\{x \mid x \neq \pm 3\}$

y-int $f(0) = \dfrac{0+0}{0-9} = 0$ $(0,0)$

x-int: $-x^3+3x^2=0$

$-x^2(x-3)=0$

$-x^2=0$ $\quad x-3=0$

$x=0$ $\quad x=3$

$(0,0)$ $\quad$ ~~x~~ not in domain!

$(0,0)$ not in domain!

$$f(x) = \frac{-x^3+3x^2}{x^2-9} = \frac{-x^2(x-3)}{(x-3)(x+3)} = \frac{-x^2}{x+3}$$

hor. asy: num > den → no hor. asymp.

vert. asy: $x+3=0$

$\boxed{x=-3}$

$x=3$ is a hole

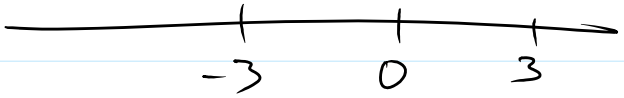
sym:

$$f(-x) = \frac{-(-x)^3+3(-x)^2}{(-x)^2-9} = \frac{+x^3+3x^2}{x^2-9}$$

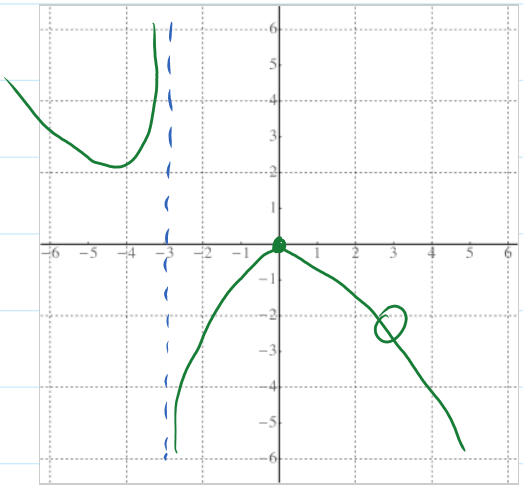
$$= \frac{-(-x^3-3x^2)}{x^2-9} = -\frac{-x^3-3x^2}{x^2-9}$$

neither

Sign chart



	$(-\infty,-3)$	$(-3,0)$	$(0,3)$	$(3,\infty)$
pt	-4	-1	1	4
$-x^2$	$-$	$-$	$-$	$-$
$x+3$	$-$	$+$	$+$	$+$
$f(x)$	$+$	$-$	$-$	$-$


6
5
4
3
2
1
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−4
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−2
−1
1
2
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4
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6
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−2
−3
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−6