Parabola

- $y = 2x^2 + 8x - 20$

Vertex: x-coord: $\frac{-b}{2a}$, $\frac{-8}{2\cdot 2} = \frac{-8}{4} = -2$

$y(-2) = 2\cdot(-2)^2 + 8(-2) - 20 = 2\cdot 4 - 16 - 20 = -28$

$\boxed{(-2, -28)}$

axis of symmetry: $\boxed{x = -2}$

y-int: $y(0) = 2\cdot 0 + 8\cdot 0 - 20 = -20$

$\boxed{(0, -20)}$

x-int: $y(x) = 0$

$2x^2 + 8x - 20 = 0$

$x^2 + 4x - 10 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4\cdot(-10)}}{2} = \frac{-4 \pm \sqrt{16 + 40}}{2}$$

$$= \frac{-4 \pm \sqrt{56}}{2} = \frac{-4 \pm \sqrt{4\cdot 14}}{2} = \frac{-4 \pm 2\sqrt{14}}{2}$$

$= -2 \pm \sqrt{14}$

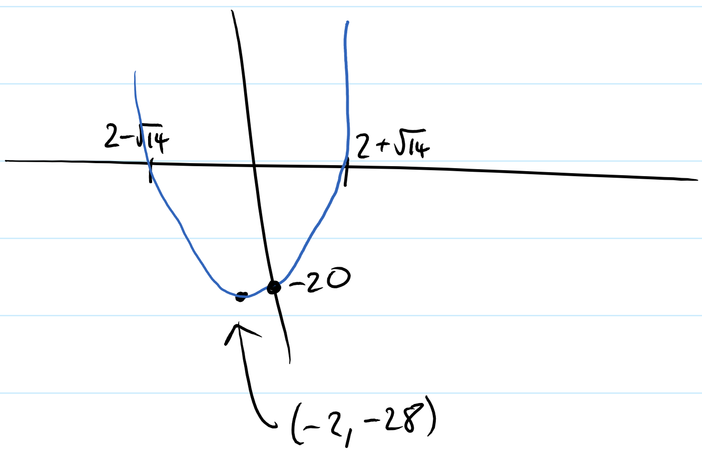
$\boxed{(-2 \pm \sqrt{14}, 0)}$

Domain: $y = 2x^2 + 8x - 20$ → parabola is open up.

$\boxed{(-\infty, \infty)}$

Range: vert: $(-2, -28)$

$\boxed{[-28, \infty)}$



Sketch $y = 9 - (x-4)^2$

$= -1(x-4)^2 + 9$

vertex: $(4, 9)$

axis of sym: $x = 4$

x-int: $y(x)=0$

$9-(x-4)^2=0$

$9=(x-4)^2$

$\pm 3 = x-4$

$4\pm 3 = x \rightarrow 7, 1$

$\boxed{\begin{matrix}(7,0)\\(1,0)\end{matrix}}$

y-int: $y(0)=9-(0-4)^2 = 9-16=-7$

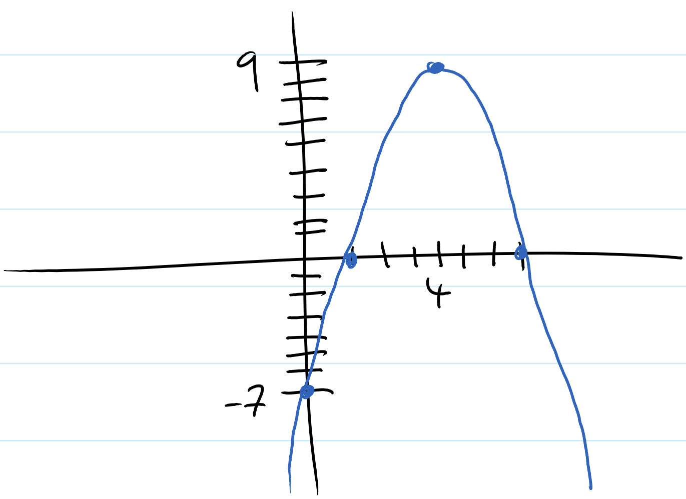
$\boxed{(0,-7)}$

Dom: $\boxed{(-\infty,\infty)}$

Range: $y=9-(x-4)^2$ opens down

vertex: $(4,9)$

$\boxed{(-\infty,9]}$



Determine the quadratic function whose graph has vertex at $(2,-1)$ and y-int $(0,-5)$.

st. eq: $y=a(x-h)^2+k$

$$\text{st. eq:} \quad y = a(x-h)^2 + k$$

$$y = a(x-2)^2 - 1$$

y-int:

$$y(0) = -5$$
$$a(0-2)^2 - 1 = -5$$
$$a \cdot 4 - 1 = -5$$
$$\frac{4a}{4} = \frac{-4}{4}$$
$$a = -1$$

$$\boxed{y(x) = -1(x-2)^2 - 1}$$

inequalities

- solve

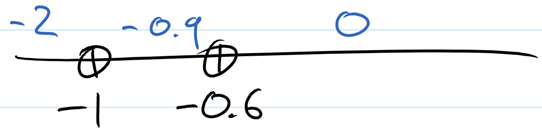
$$5x < -2 - 3x^2$$
$$+2+3x^2 \qquad +2+3x^2$$

$$3x^2 + 5x + 2 < 0$$

solve:

$$3x^2 + 5x + 2 = 0$$
$$3x^2 + 3x + 2x + 2 = 0$$
$$3x(x+1) + 2(x+1) = 0$$
$$(x+1)(3x+2) = 0$$
$$\underline{x = -1} \qquad \underline{x = -\tfrac{2}{3}} \approx -0.\overline{66}$$

$$A \cdot C = 3 \cdot 2 = 6$$
$$A + C = 5$$



	$(-\infty,-1)$	$(-1,-\frac{2}{3})$	$(-\frac{2}{3},\infty)$
	-2	-0.9	0
$x+1$	$-$	$+$	$+$
$3x+2$	$-$	$-$	$+$
$f(x)$	$+$	$-$	$+$

solution: $\boxed{(-1,-\frac{2}{3})}$

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solve: $$\frac{-x+1}{x-7} \geq 0$$

$-x+1=0$ $\qquad$ $x-7=0$

$\underline{x=1}$ $\qquad$ $\underline{x=7}$

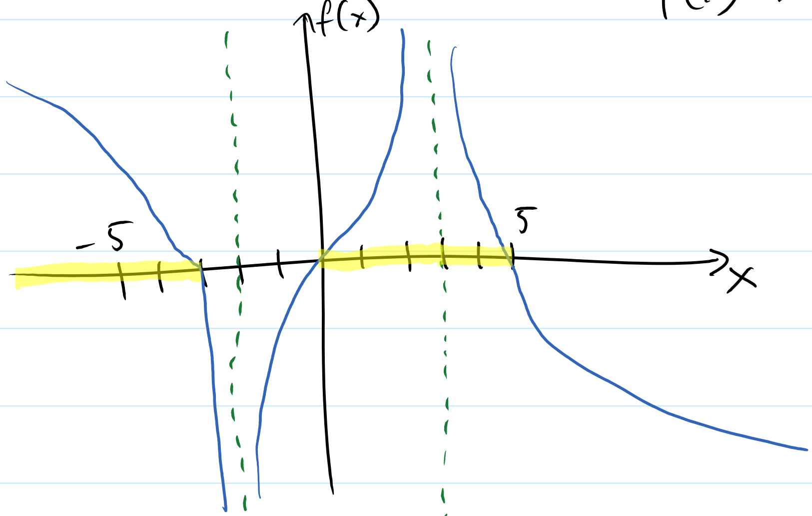
	$(-\infty,1]$	$[1,7)$	$(7,\infty)$
pt	0	2	8
$-x+1$	$+$	$-$	$-$
$x-7$	$-$	$-$	$+$
$f(x)$	$-$	$+$	$-$

$x-7$	$-$	$-$	$+$
$f(x)$	$-$	$+$	$-$

solution: $\boxed{[1, 7)}$

Use the graph below to solve:

$f(x) \geq 0$



solution: $(-\infty, -3] \cup \cancel{[0, 5]}$

$$\boxed{(-\infty, -3] \cup [0, 3) \cup (3, 5]}$$

x = 3 has to excluded.
since it's a vertical asym.

Rational functions

Graph: $y = \dfrac{x^3 - 4x}{x^2 - 1}$

Domain: $x^2 - 1 \neq 0$

$x^2 \neq 1$

$\boxed{x \neq \pm 1}$

$$\boxed{\begin{array}{c}(-\infty, -1) \cup (-1, 1) \cup (1, \infty) \\ \{x \mid x \neq \pm 1\}\end{array}}$$

$\boxed{x \neq \pm 1}$

$$y = \frac{x^3-4x}{x^2-1} = \frac{x(x^2-4)}{(x-1)(x+1)} = \frac{x(x-2)(x+2)}{(x-1)(x+1)}$$

x-int: $x(x-2)(x+2) = 0$

$x=0 \qquad x=2 \qquad x=-2$

$\boxed{(0,0),\ (\pm 2,0)}$

y-int: $y(0) = \dfrac{0(0-2)(0+2)}{(0-1)(0+1)} = 0$

$\boxed{(0,0)}$

vert. asymptote(s): $(x-1)(x+1) = 0$

$\boxed{x = \pm 1}$

hor. asymptote: none b/c degree of num is greater than the deg of denom.

Symmetries:

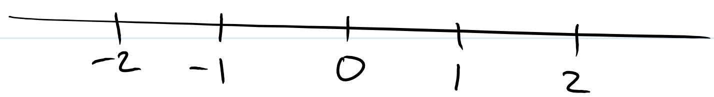
$$f(-x) = \frac{(-x)^3-4(-x)}{(-x)^2-1} = \frac{-x^3+4x}{x^2-1} \quad \text{not even}$$

$$= \frac{-(x^3-4x)}{x^2-1} = -\frac{x^3-4x}{x^2-1} \quad \text{odd}$$

$$= \frac{-(x^3-4x)}{x^2-1} = -\frac{x^3-4x}{x^2-1}$$ odd

the graph is symmetric with respect to the origin.

sign chart:



	$(-\infty,-2)$	$(-2,-1)$	$(-1,0)$	$(0,1)$	$(1,2)$	$(2,\infty)$
pt	-3	-1.5	$-.5$	$.5$	1.5	3
x	$-$	$-$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	$+$	$+$
$x+1$	$-$	$-$	$+$	$+$	$+$	$+$
$f(x)$	$-$	$+$	$-$	$+$	$-$	$+$