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Thursday, November 16, 2017 3:55 PM

Section 2.6 & 2.7

function composition

A composition of the function f with g is denoted by $(f \circ g)$ and it is defined as

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x such that

1) x is in the domain of g
2) $g(x)$ is in the domain of f.

Ex: Given $f(x) = 3x - 4$, $g(x) = x^2 - 2x + 6$

find:

- $(f \circ g)(x) = f(g(x)) = f(x^2 - 2x + 6)$

$$= 3(x^2 - 2x + 6) - 4$$

$$= 3x^2 - 6x + 18 - 4 = \boxed{3x^2 - 6x + 14}$$

Dom: $(-\infty, \infty)$

- $(g \circ f)(x) = g(f(x)) = g(3x - 4)$

$$= (3x-4)^2 - 2(3x-4) + 6$$

$\cdot (g \circ f)(x) = g(f(x)) = g(3x-4)$

$$= (3x-4)^2 - 2(3x-4) + 6$$
$$= 9x^2 - 24x + 16 - 6x + 8 + 6$$
$$= \boxed{9x^2 - 30x + 30}$$

Dom: $(-\infty, \infty)$

Ex:

$$f(x) = \frac{2}{x-1}, \quad g(x) = \frac{3}{x}$$

$\cdot \ (f \circ g)(x) = f(g(x)) = f\left(\frac{3}{x}\right) = \dfrac{2}{\frac{3}{x} - 1}$

$$= \frac{2}{\frac{3}{x} - \frac{1}{1}} = \frac{2}{\frac{3-x}{x}} = \frac{2}{1} \cdot \frac{x}{3-x}$$

$$= \boxed{\frac{2x}{3-x}}$$

Dom: $\{x \mid x \neq 3, 0\}$

$x \neq 3$ b/c $x \neq 0$

$\cdot (g \circ f)(x) = g(f(x)) = g\left(\frac{2}{x-1}\right) = \dfrac{3}{\frac{2}{x-1}}$

$$= \frac{3}{1} \cdot \frac{x-1}{2} = \boxed{\frac{3(x-1)}{2}}$$

Dom: $\{x \mid x \neq 1\}$

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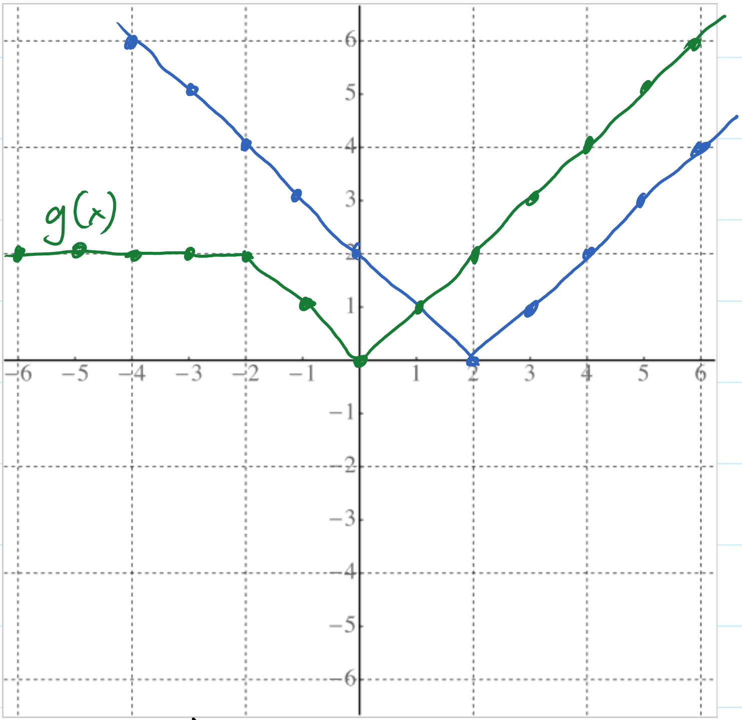


find:

$$(f \circ g)(0) = f(g(0)) = f(0) = \boxed{2}$$

$$(g \circ f)(0) = g(f(0)) = g(2) = \boxed{2}$$

$f(g(-2)) = f(2) = \boxed{0}$ | $g(f(-2)) = g(4) = \boxed{4}$

Decomposition of a function

Ex: Express $h(x)$ as a composition of two functions:

- $h(x) = \sqrt[3]{x^2+1}$ $h(x) = (f \circ g)(x)$

$g(x) = x^2+1$, $f(x) = \sqrt[3]{x}$

- $h(x) = \dfrac{1}{\sqrt{x-2}}$

$g(x) = x-2$, $f(x) = \dfrac{1}{\sqrt{x}}$

$\cdot\ h(x) = \sqrt{2x-x^2} \cdot \frac{3}{(2x-x^2)^3}$

$g(x) = 2x - x^2$, $f(x) = \sqrt{x} \cdot \frac{3}{x^3}$

Section 2.7

find $(f \circ g)(x)$ and $g(f(x))$ of:

$$g(x) = \frac{x-2}{3} \qquad f(x) = 3x+2$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x-2}{3}\right)$$
$$= \cancel{3} \cdot \frac{x-2}{\cancel{3}} + 2 = x - 2 + 2 = \boxed{x}$$

$$g(f(x)) = g(3x+2) = \frac{3x+2-2}{3} = \frac{3x}{3} = \boxed{x}$$

Def: Given two functions $f(x)$ and $g(x)$, if $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, then we say that g is an inverse of f or f is an inverse of g.

Ex: Are $f(x) = 4x - 7$ and $g(x) = \frac{x+7}{4}$ inverses?

$$f(g(x)) = f\left(\frac{x+7}{4}\right) = 4 \cdot \frac{x+7}{4} - 7 = \boxed{x} \checkmark$$

$$g(f(x)) = g(4x-7) = \frac{4x-7+7}{4} = \boxed{x} \checkmark$$

$\boxed{\text{Yes}}$

To find an inverse of a function $y = f(x)$, replace "x" with "y" and "y" with "x" and solve the equation for y.

- Find the inverse $y = 4x - 7$

$$x = 4y - 7$$

solve

$$x + 7 = 4y$$

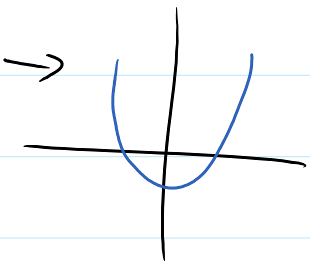
$$\frac{x+7}{4} = \frac{4y}{4}$$

$$\boxed{y = \frac{x+7}{4}}$$

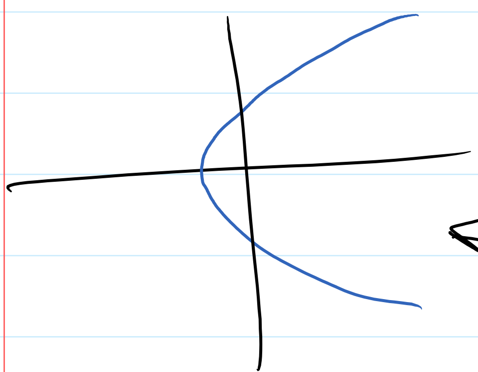
- Find the inverse for $y = x^3 - 1$

$$x = y^3 - 1$$
$$+1 \qquad +1$$
$$y^3 = x+1$$
$$\boxed{y = \sqrt[3]{x+1}}$$

• Find the inverse of $y = x^2 - 1$



$$x = y^2 - 1$$



$$y^2 = x+1$$
$$y = \pm\sqrt{x+1}$$

Not a function

Therefore $y = x^2 - 1$ has no inverse.

Notation

The inverse function of $f(x)$, if it exists, is denoted as $f^{-1}(x)$

Horizontal line test

A function f has an inverse, f^{-1}, if there is no horizontal line that intersects the graph of f at more than one point.

Notation

An one-to-one function is a function with an inverse.

↳ Is $f(x) = x^3 - 1$ one-to-one? Yes

Is $f(x) = x^2 - 1$ one-to-one? NO

Is $g(x) = \dfrac{x-2}{x-1}$ an one-to-one function?

$$y = \frac{x-2}{x-1}$$

$$\updownarrow$$

$$x = \frac{y-2}{y-1}$$

since the graph is symmetric with respect to the line $y = x$, the inverse is the same.

$$(y-1)x = y-2$$

$$yx - x = y - 2$$

$$-y \qquad -y$$

$$+x \qquad +x$$

$$yx - y = x - 2$$

$$\frac{y(x-1)}{x-1} = \frac{x-2}{x-1}$$

$$\boxed{y = \frac{x-2}{x-1}}$$

Find g^{-1} of $g(x) = \frac{2}{x-3}$.

Find the domain & range of g and g^{-1}.

$$y = \frac{2}{x-3} \longleftrightarrow x = \frac{2}{y-3}$$

$$(y-3)x = 2$$

$$yx - 3x = 2$$

$$\frac{yx}{x} = \frac{2+3x}{x}$$

$$y = \frac{2+3x}{x}$$

$$\boxed{g^{-1}(x) = \frac{2+3x}{x}}$$ Dom: $\{x \mid x \neq 0\}$ Range: $\{x \mid x \neq 3\}$

$$g(x) = \frac{2}{x-3}$$ Dom: $\{x \mid x \neq 3\}$ Range: $\{x \mid x \neq 0\}$

The domain of f is the range of f^{-1}.

The domain of f is the range of f^{-1}.
The domain of f^{-1} is the range of f.

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let $f(x) = \sqrt{2x-3}$, $g(x) = -x$

$$(f \circ g)(x) = f(g(x)) = f(-x) = \sqrt{2(-x)-3}$$
$$= \boxed{\sqrt{-2x-3}}$$

$$(g \circ f)(x) = g(\sqrt{2x-3}) = \boxed{-\sqrt{2x-3}}$$
