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Tuesday, November 21, 2017 4:59 PM

Section 4.1

Def: The exponential function f with base b is defined by

$$f(x) = b^x \quad \text{or} \quad y = b^x,$$

where b is a positive constant $\neq 1$.

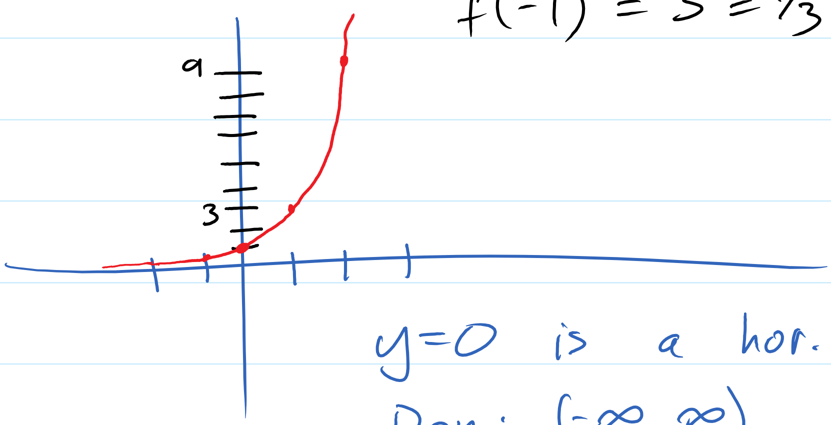
Ex:

$$f(x) = 3^x$$

Find:

$$f(2) = 3^2 = 9$$
$$f(3) = 3^3 = 27$$
$$f(1) = 3^1 = 3$$
$$f(0) = 3^0 = 1$$
$$f(-1) = 3^{-1} = 1/3$$

$$\boxed{x^{-a} = \frac{1}{x^a} \qquad \left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a}$$



$y=0$ is a hor. asymptote.

Dom: $(-\infty, \infty)$

Range: $(0, \infty)$

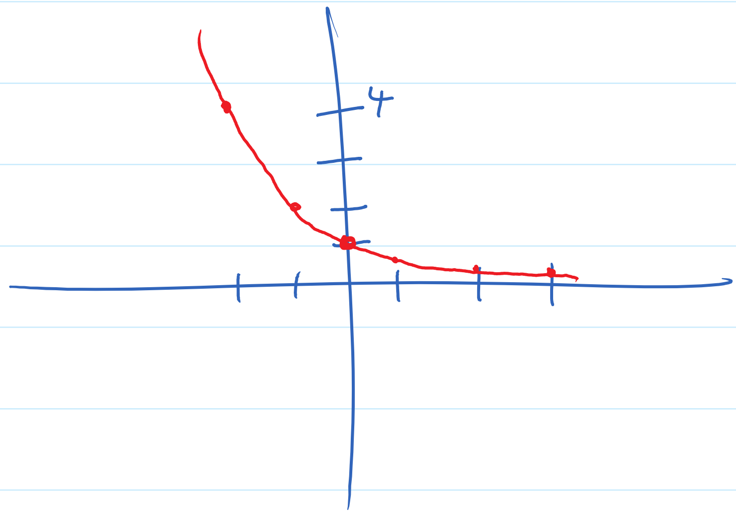
E $f(x) = \left(\frac{1}{\ }\right)^x$

Ex: $f(x) = \left(\frac{1}{2}\right)^x$

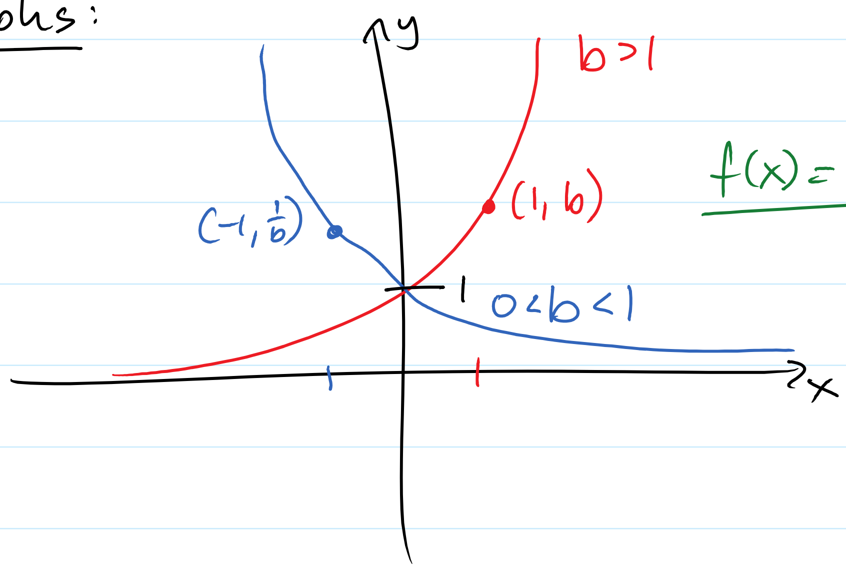
x	-2	-1	0	1	2	3
$f(x)$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

$\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = 2^2 = 4$

$\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$



Graphs:



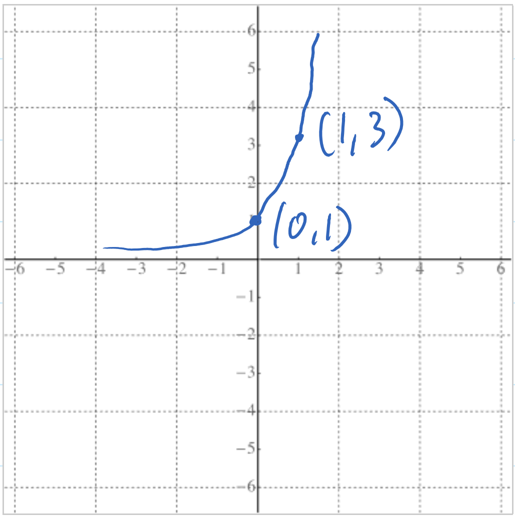
$f(x) = b^x$

Ex: Use the graph of $f(x) = 3^x$ to graph $g(x) = 3^{x+1}$

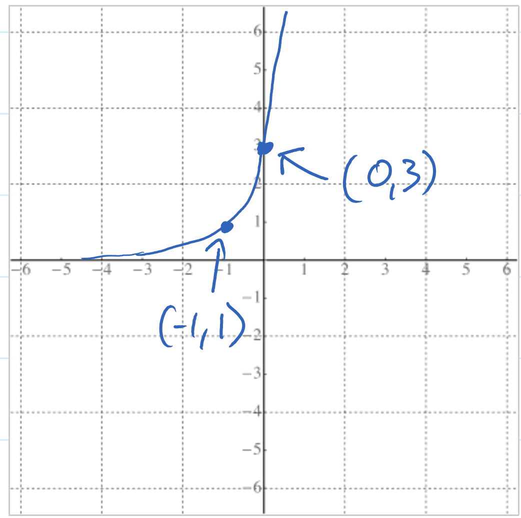
$g(x) = 3^{x+1}$

hor. shift left by 1

3^x



3^{x+1}



Transformations:

- horizontal stretch/ shrink $f(x) = b^{a \cdot x}$

$$\cdot\ 2^x \xrightarrow{\text{shrink}} 2^{3x}$$

$$\cdot\ \left(\tfrac{1}{2}\right)^x \xrightarrow{\text{hor. stretch}} \left(\tfrac{1}{2}\right)^{x/3}$$

- hor. shift by a : $f(x) = b^{x \pm a}$

- ver. shift by a: $f(x) = b^x \pm a$

- ver. stretch/ shrink by a: $f(x) = a \cdot b^x$

hor. shift

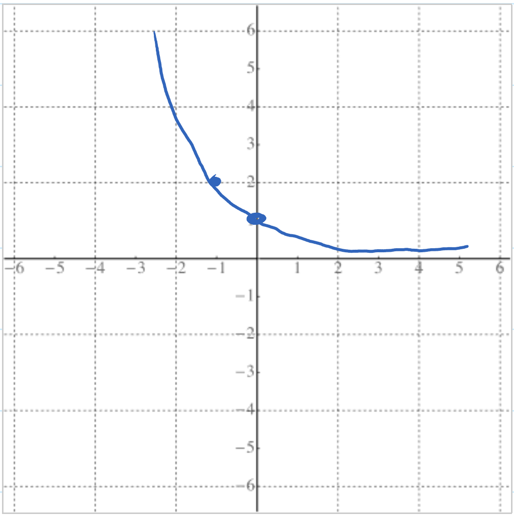
Ex:

Use the graph $f(x)=\left(\frac{1}{2}\right)^{x}$ to graph $g(x)=3\left(\frac{1}{2}\right)^{x-2}+1$

ver. stretch

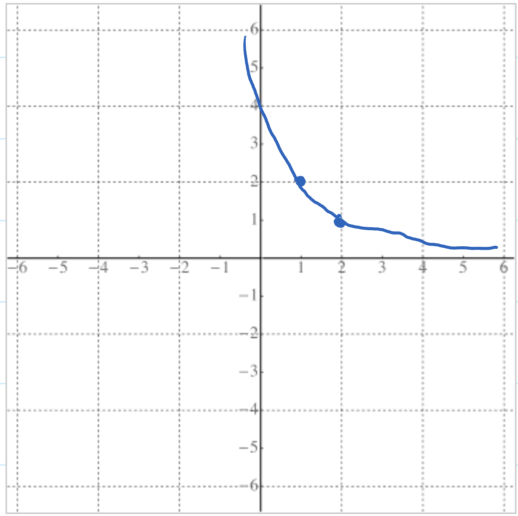
ver. shift

$\left(\frac{1}{2}\right)^{x}$



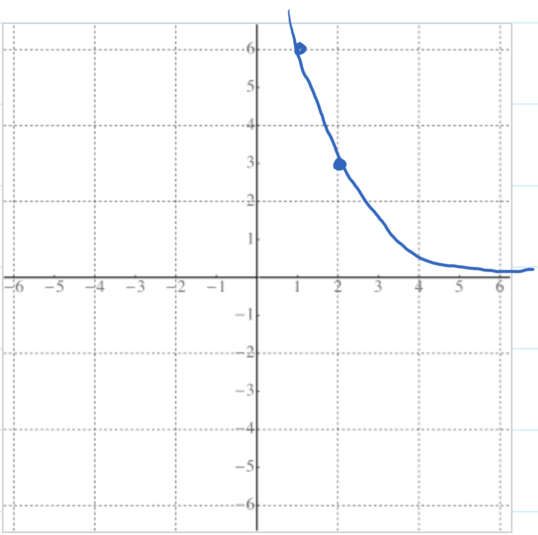
$\longrightarrow$

$\left(\frac{1}{2}\right)^{x-2}$



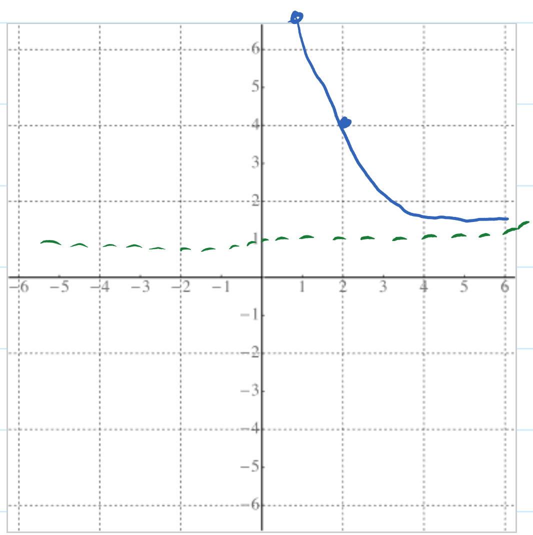
$\swarrow$

$3\left(\frac{1}{2}\right)^{x-2}$



$\longrightarrow$

$3\left(\frac{1}{2}\right)^{x-2}+1$



The Euler constant: $\boxed{e} \approx 2.718\ldots$

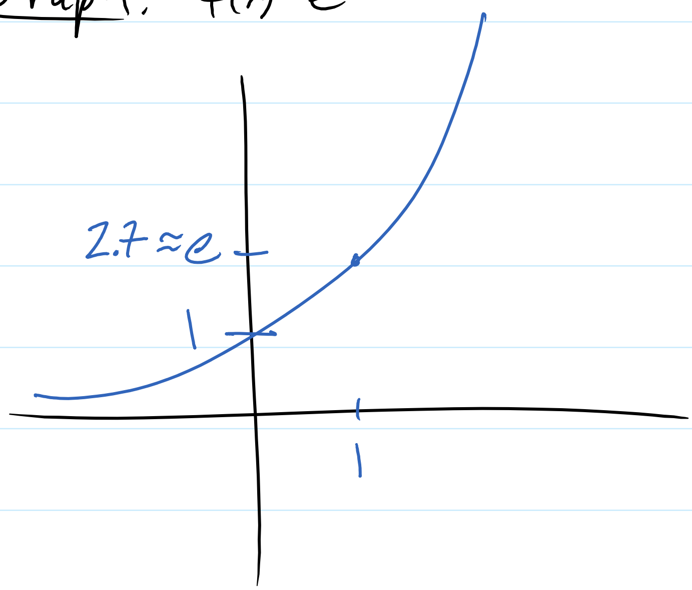
e is the number you get if $x\to\infty$ of

$f(x)=\left(1+\frac{1}{x}\right)^{x}$

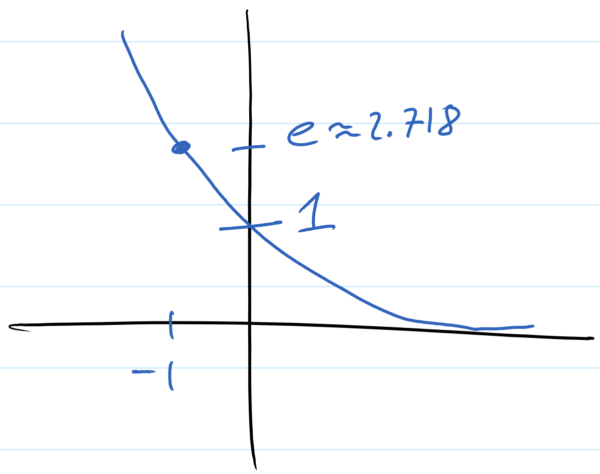
or

$$f(x) = \left(1+\frac{1}{x}\right)^x$$

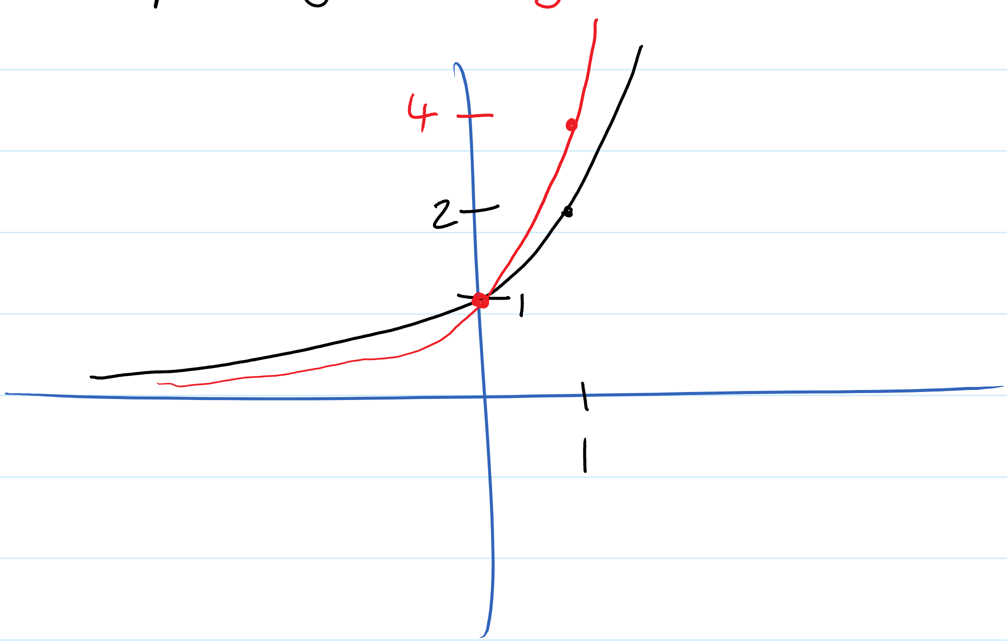
Graph: $f(x)=e^x$



$f(x)=e^{-x}$



Ex: Graph: $y=2^x$, $y=4^x$



Section 4.4 (Algebra with exponential equation)

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Rule:

If $b^M = b^N$, then $M = N$.

Solve:

- $2^x = 2^4 \longrightarrow x = 4$

- $27^{x+3} = 9^{x-1}$

$$\boxed{(x^a)^b = x^{a \cdot b}}$$

$$(3^3)^{x+3} = (3^2)^{x-1}$$

$$3^{3x+9} = 3^{2x-2} \longrightarrow 3x+9 = 2x-2$$

$$3x - 2x = -2 - 9$$

$$\boxed{x = -11}$$

- $2^{3x-8} = 16$

$$2^{3x-8} = 2^4 \longrightarrow 3x-8 = 4$$

$$3x = 12$$

$$\boxed{x = 4}$$

$3x-6$

- $5^{3x-6} = 125$

$5^{3x-6} = 5^3 \rightarrow 3x-6=3$

$3x = 9$

$\boxed{x=3}$

- $\left(\frac{1}{2}\right)^{2x+1} = 8$

$(2^{-1})^{2x+1} = 2^3$

$2^{-(2x+1)} = 2^3 \longrightarrow -(2x+1) = 3$

$-2x-1=3$

$-2x = 4$

$x = -\frac{4}{2} = \boxed{-2}$