11/28

Tuesday, November 28, 2017 5:02 PM

Section 4.2 (4.3)

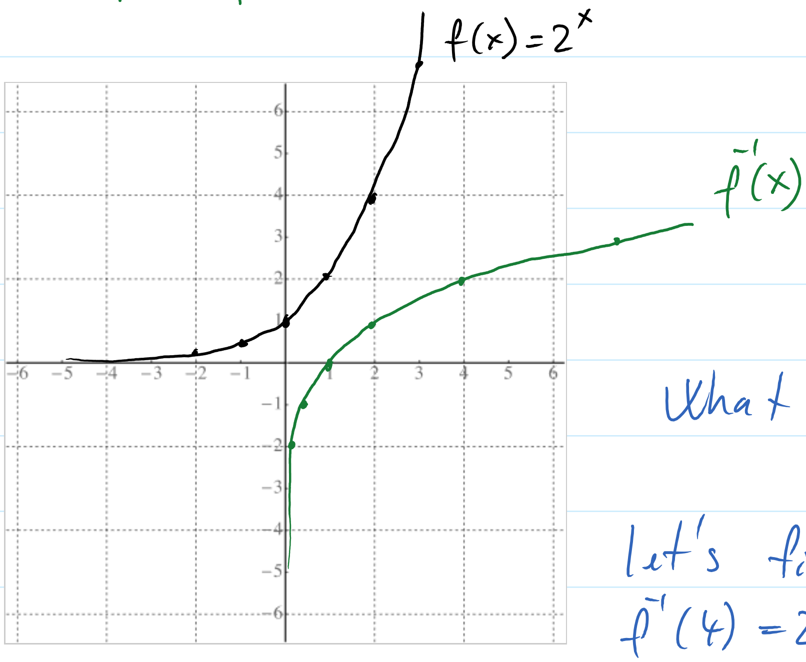
x	-2	-1	0	1	2	3	4
2^x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

← exponential function $f(x)=2^x$ (Section 4.1)

what is the inverse of $f(x)=2^x$?

The inverse is $g(x)$.

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16
$f^{-1}(x)=g(x)$	-2	-1	0	1	2	3	4



What is $f^{-1}(3)$.

let's find out, why/how
$f^{-1}(4)=2$

$2^x=y \longrightarrow 2^2=4$

$\downarrow$

$2^y=x$

inverse
$x=4, \quad y=2$

$\boxed{f^{-1}(3)} \to 2^{\boxed{?}} = 3$

Def: For $x > 0$ and $b > 0$, $b \neq 1$

$y = \log_b x$ is equivalent to $b^y = x$.

The function $f(x) = \log_b x$ is the logarithm function with base b.

logorithmic form: $y = \log_b x$ } exponential form: $b^y = x$

Ex: Write in exp. form:

- $\log_5 x = 2$
  $2 = \log_5 x$ } $5^2 = x$

- $\log_b 64 = 3 \iff b^3 = 64$

- $\log_3 7 = y \iff 3^y = 7$

$\log_{12} x = 2 \Leftarrow 12^2 = x$

$\log_b 8 = 3 \Leftarrow b^3 = 8$

$\log_b 8 = 3 \quad \Leftarrow \quad b^3 = 8$

$\log_e 9 = y \quad \Leftarrow \quad e^y = 9$

Ex: Evaluate:

- $\log_2 16 = y \longrightarrow 2^y = 16$

find y

$y = 4$

$\log_2 16 = \boxed{4}$

- $\log_7\left(\frac{1}{49}\right) = y \rightarrow 7^y = \frac{1}{49}$

$7^y = \frac{1}{7^2}$

$\log_7\left(\frac{1}{49}\right) = \boxed{-2}$

$7^y = 7^{-2}$

$y = -2$

- $\log_{25}(5) = y \longrightarrow 25^y = 5$

$(5^2)^y = 5$

$5^{2y} = 5$

$\log_{25} 5 = \boxed{\frac{1}{2}}$

$2y = 1$

$\boxed{y = 1/2}$

- $\log_2 \sqrt[5]{2} = y \longrightarrow 2^y = 2^{1/5}$

- $\log_2 \sqrt[5]{2} = y \longrightarrow \quad 2^y = 2^{1/5}$

$y = 1/5$

$\log_2 \sqrt[5]{2} = \boxed{\frac{1}{5}}$

- $\log_b 1 = y \longrightarrow \quad b^y = 1$

$\underline{y = 0}$

$\boxed{\log_b 1 = 0} \longleftrightarrow \boxed{b^0 = 1}$

- $\log_b b = y \longrightarrow \quad b^y = b$

$y = 1$

$\boxed{\log_b b = 1} \longleftrightarrow \boxed{b^1 = b}$

Properties of $f(x) = \log_b(x)$

1) inverse of $y = b^x$

Domain of b^x: $(-\infty, \infty)$	Range of b^x: $(0, \infty)$
Domain of $\log_b x$ $(0, \infty)$	Range of $\log_b x$: $(-\infty, \infty)$

Domain of $\log_b x$: $(0, \infty)$

Range of $\log_b x$: $(-\infty, \infty)$

2) $f(f^{-1}(x)) = \boxed{\log_b(b^x) = x}$

$\log_3(3^1) = 1$

$\log_3(3^2) = \log_3 9 = y \rightarrow 3^y = 9$

$3^y = 3^2$

$y = 2$

$\log_3(3^2) = 2$

$\log_3(3^3) = y \rightarrow 3^y = 3^3$

$y = 3$

$\log_3(3^3) = 3$

$f^{-1}(f(x)) = \boxed{b^{\log_b x} = x}$

Evaluate:

- $\log_7 7 = 1$

identities / formulas

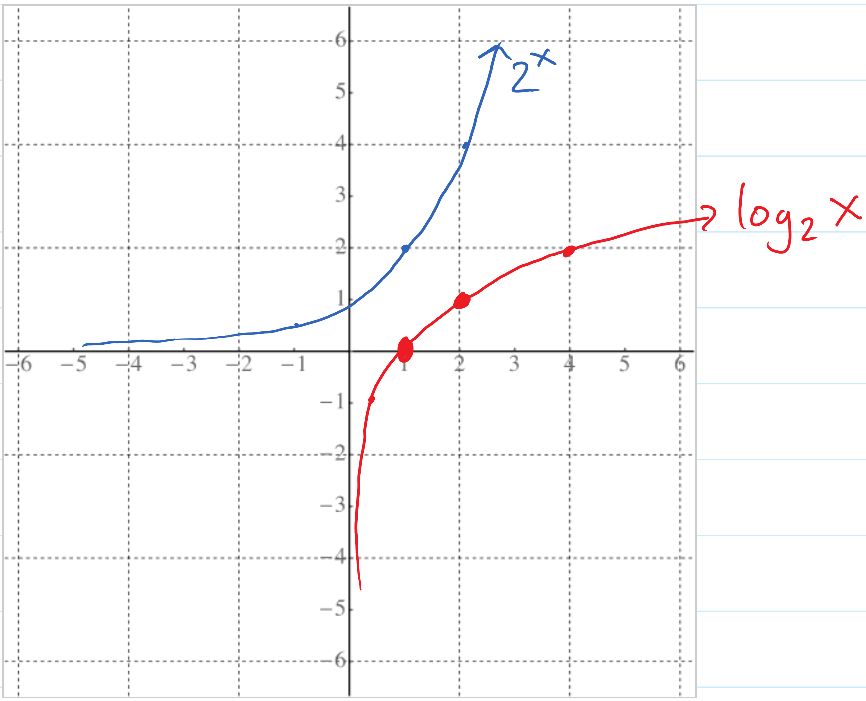
- $\log_b b = 1$
- $\log_b 1 = 0$

x

- $\log_5 1 = 0$
- $\log_4 4^5 = 5$
- $6^{\log_6 9} = 9$

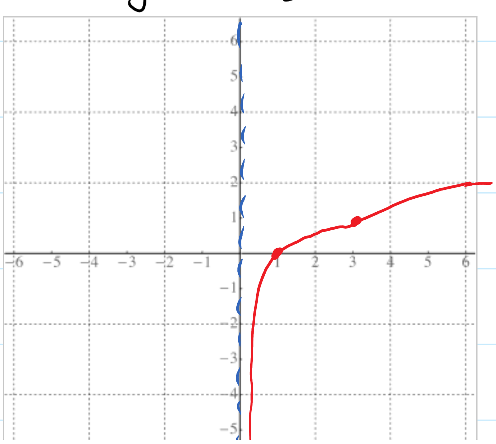
- $\log_b b^x = x$
- $b^{\log_b x} = x$

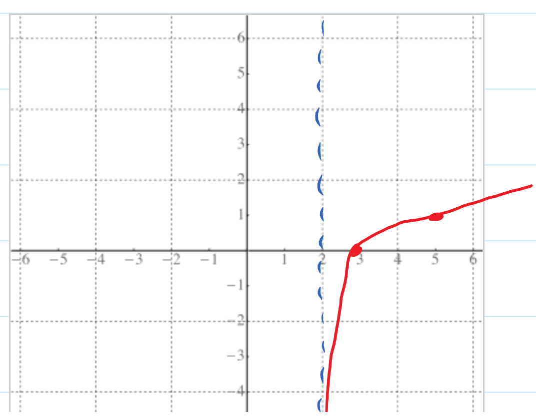
Graphs:

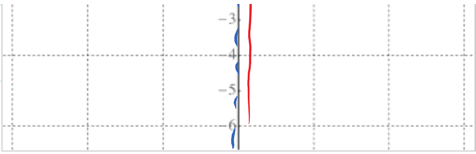


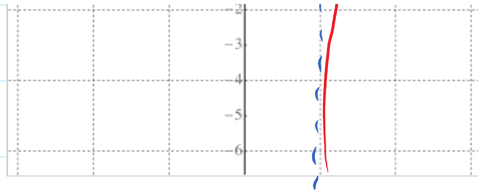
Graph: $y = \log_3 (x-2)$

$y = \log_3 x \xrightarrow{\text{hor. shift by } 2} y = \log_3 (x-2)$









what is the domain of $\log_3(x-2)$

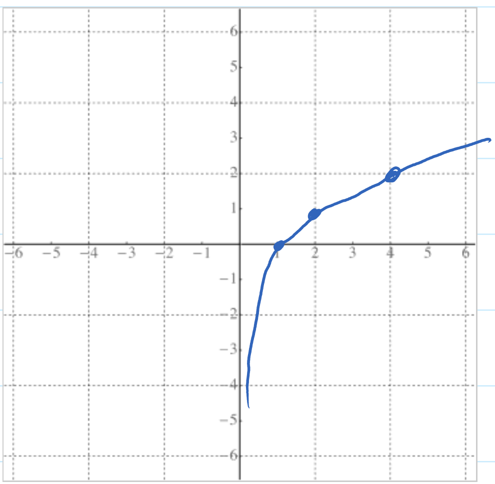
$$\boxed{(2,\infty)}$$

$x-2>0$

$x>2$

graph: $y=-\log_2\left(\frac{1}{3}x\right)+2$

$y=\log_2 x$



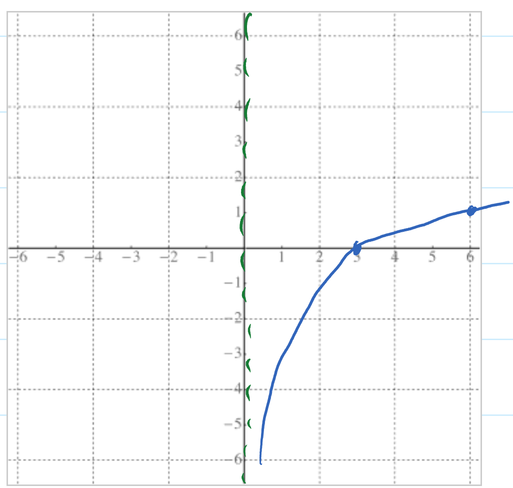
hor. stretch

additionally:

$\log_b(x^a)=a\cdot\log_b(x)$

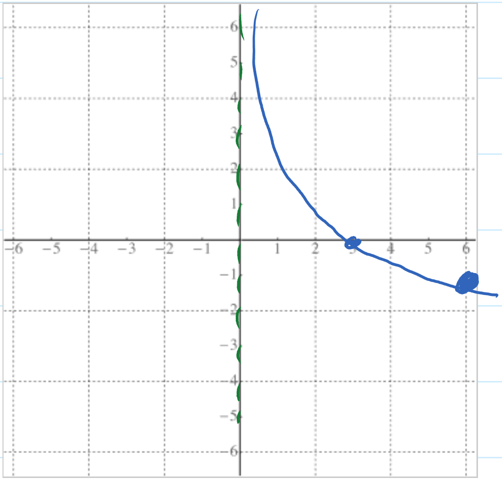
(next week)

$y=\log_2\left(\frac{1}{3}x\right)$



reflection about x-axis

$y=-\log_2\left(\frac{1}{3}x\right)$



vert. shift
by 2 up

$y = -\log_2\left(\frac{1}{3}x\right) + 2$