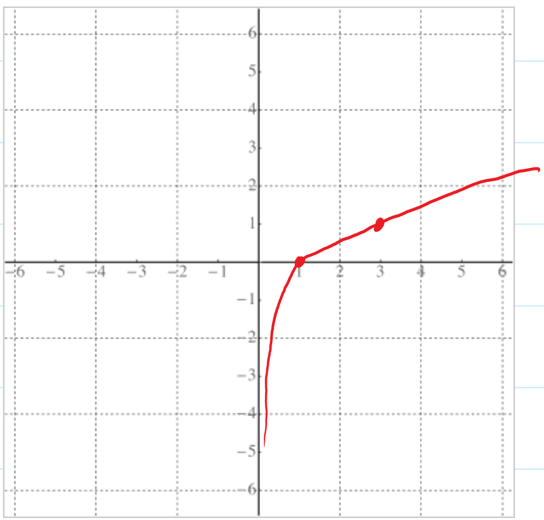
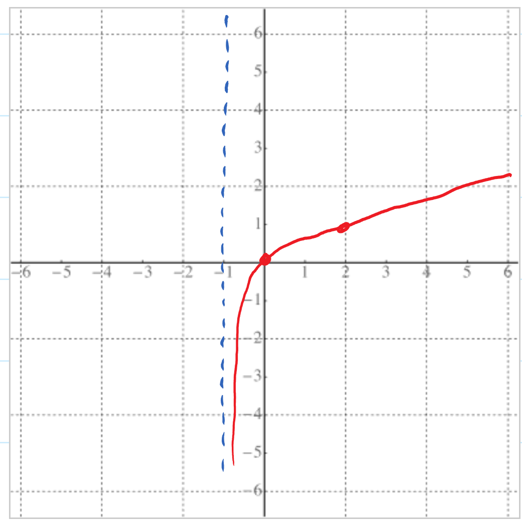
Graph: $-2\log_3(x+1)$

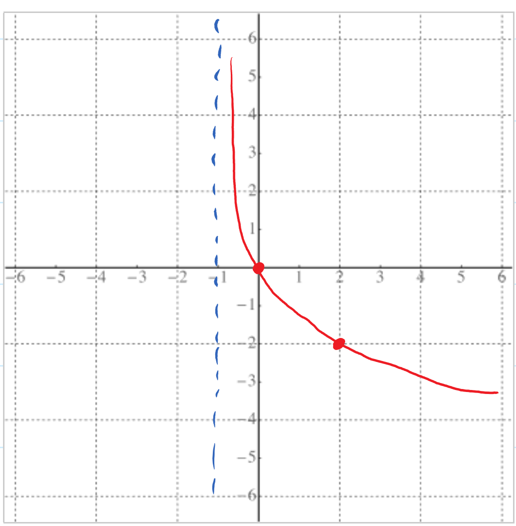
base: $\log_3 x$ $\xrightarrow{\text{left by 1}}$ $y = \log_3(x+1)$





$y = -2\log_3(x+1)$

- reflection about the x-axis
- vert. stretch by 2



Order of trans:

1) horizontal shift
2) hor./vertical stretch/shrink reflection
3) vertical shift

Graph: $\log_{\frac{1}{2}}\left(-\frac{1}{3}x+2\right)-1$

$\log_{\frac{1}{2}}(2) = y$

Graph: $\log_{\frac{1}{2}}\left(-\frac{1}{3}x+2\right)-1$

$\log_{\frac{1}{2}}(2)=y$

$\Updownarrow$

$\left(\frac{1}{2}\right)^y=2$

$\left(2^{-1}\right)^y=2$

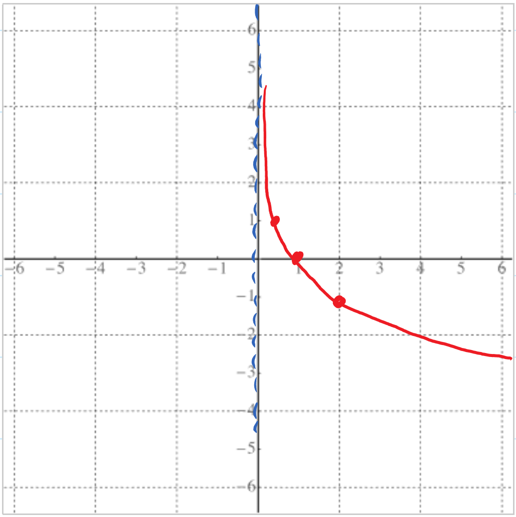
$2^{-y}=2$

$\downarrow$

$-y=1$

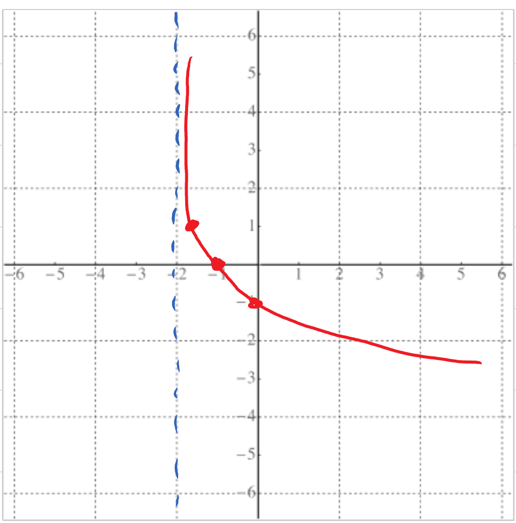
$\underline{y=-1}$

base: $\log_{\frac{1}{2}}(x)$



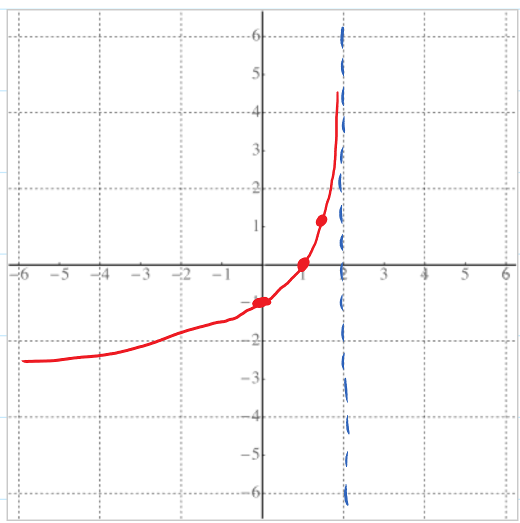
left by 2

$\log_{\frac{1}{2}}(x+2)$



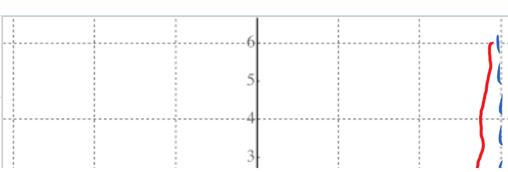
reflection about y-axis $\longrightarrow$

$\log_{\frac{1}{2}}(-x+2)$



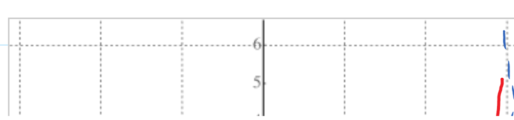
hor. stretch by 3

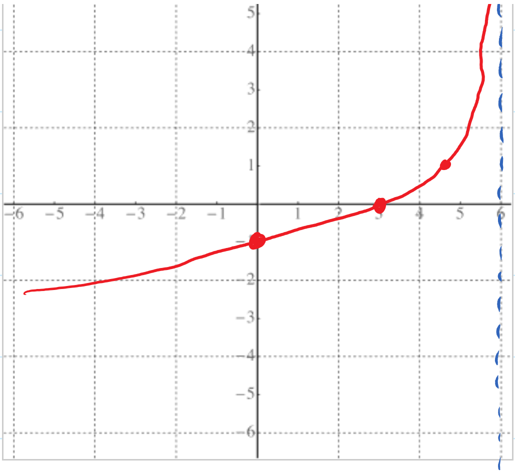
$\log_{\frac{1}{2}}\left(-\frac{1}{3}x+2\right)$



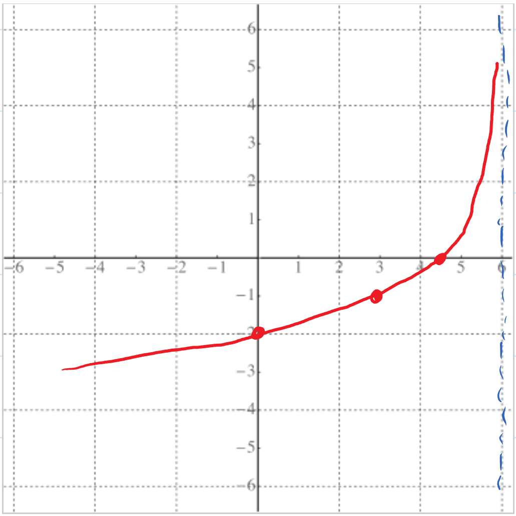
$\longrightarrow$ vertical shift down 1

$\log_{\frac{1}{2}}\left(-\frac{1}{3}x+2\right)-1$





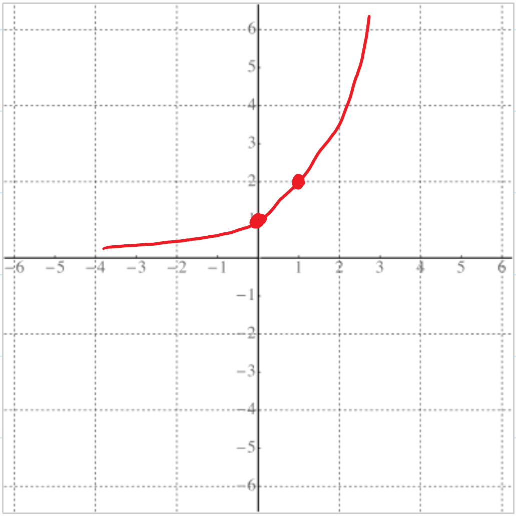
vertical shift down 1



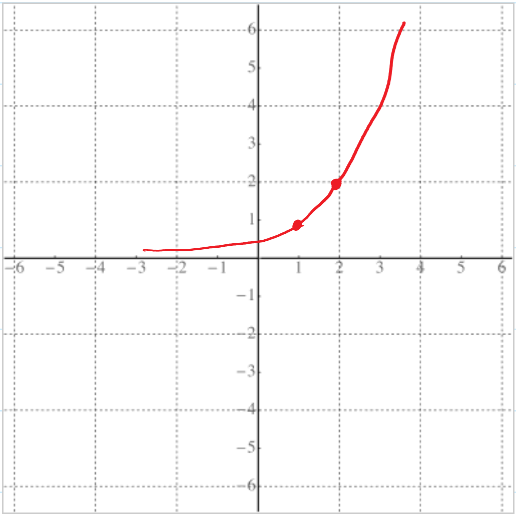
Graph: $-3\cdot 2^{\frac{1}{2}x-1}$

base: 2^{x}

2^{x-1}



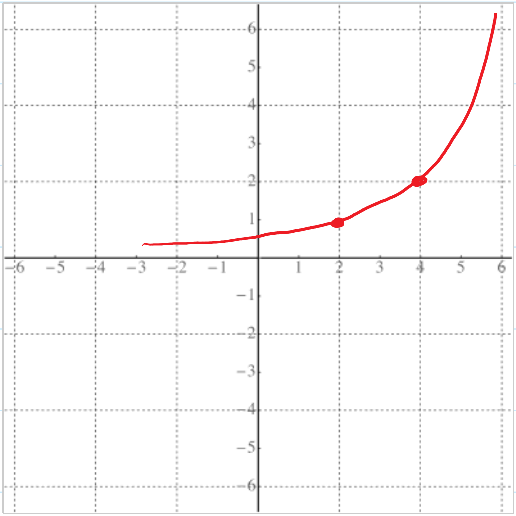
hor. shift right by 1



$2^{\frac{1}{2}x-1}$

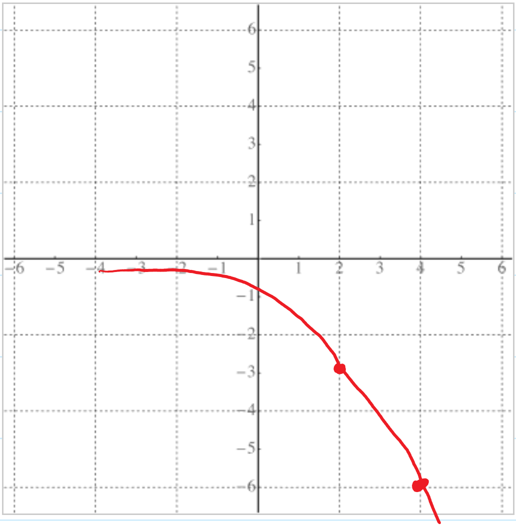
hor. stretch by 2

$-3\cdot 2^{\frac{1}{2}x-1}$



$\longrightarrow$

- reflection about the x-axis
- vertical stretch by 3



Section 4.2

Ex: Find the domain:

- $\log_4(x+3) \longrightarrow$

$$x+3>0$$
$$-3 \quad -3$$
$$x>-3$$

$$\boxed{(-3, \infty)}$$

Notation:

The common logarith is the logarithm with the base = 10, denoted as

$$\log(x) = \log_{10}(x)$$

The natural logarithm,

$$\ln(x) = \log_e(x)$$

Find the domain:

- $\ln(3-x) \rightarrow 3-x>0$

$$3>x$$

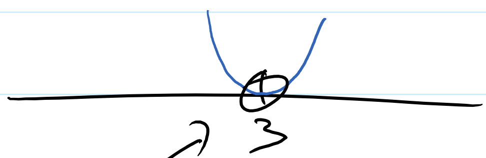
$$\boxed{(-\infty, 3)}$$

- $\ln(x-3)^2 = \ln\left[(x-3)^2\right] \rightarrow (x-3)^2>0$

$$(x-3)(x-3)>0$$

$$x-3=0$$

$$\underline{x=3}$$



exclude $\underline{x=3}$.

$$\boxed{\begin{array}{l}(-\infty, 3)\cup(3,\infty)\\ \{x \mid x\neq 3\}\end{array}}$$

- $f(x)=\log\left(\dfrac{x-2}{x+5}\right)$

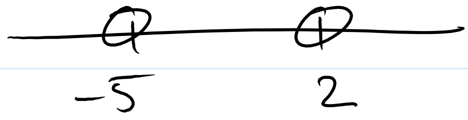
$$\frac{x-2}{x+5}>0$$

$$x-2=0 \qquad x+5=0$$

$$\underline{x=2} \qquad \underline{x=-5}$$

	$(-\infty,-5)$	$(-5,2)$	$(2,\infty)$
pt	-6	0	3

	$(-\infty,-5)$	$(-5,2)$	$(2,\infty)$
pt	-6	0	3
$x-2$	$-$	$-$	$+$
$x+5$	$-$	$+$	$+$
sign	$+$	$-$	$+$

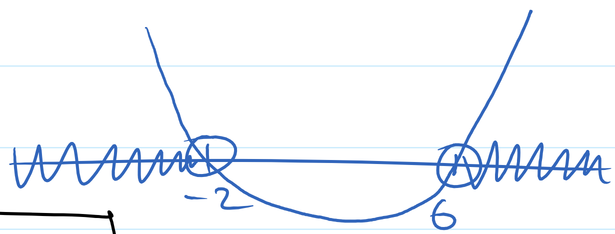


Domain: $\boxed{(-\infty,-5)\cup(2,\infty)}$

• $g(x)=\ln(x^2-4x-12) \rightarrow x^2-4x-12>0$

$(x+2)(x-6)>0$

$x=-2,\ 6$



$\boxed{\text{Domain: } (-\infty,-2)\cup(6,\infty)}$

Section: 4.3

Formulas for logs

let $M>0$, $N>0$, $b>0$, $b\neq 1$

1) $\log_b(M\cdot N)=\log_b M+\log_b N \Big\}\ b^M\cdot b^N=b^{M+N}$

2) $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ } $\frac{b^M}{b^N} = b^{M-N}$

3) $\log_b(M^p) = p \cdot \log_b M$ } $(b^M)^P = b^{M \cdot P}$

Ex: Simplify as sum of two terms:

- $\log_4(7 \cdot 5) = \log_4 7 + \log_4 5$
- $\log_4(12) = \log_4(4 \cdot 3) = \log_4 4 + \log_4 3$
  $= \boxed{1 + \log_4 3}$
- $\log(10 \cdot x) = \log 10 + \log x = \boxed{1 + \log x}$

Ex: expand each log.

- $\log_7\left(\frac{49}{x}\right) = \log_7 49 - \log_7 x = \boxed{2 - \log_7 x}$
- $\ln\left(\frac{6}{e^3}\right) = \ln 6 - \ln e^3 = \ln 6 - 3\ln e$
  $= \boxed{\ln(6) - 3} = \boxed{-3 + \ln 6}$

Note:

<u>Note:</u>

$$\log_b(x+y) \neq \log_b(x) + y$$

$$\log_b(x+y) \neq \log_b x + \log_b y$$

$$\log_2(1+1) = \log_2 2 = \underline{\underline{1}} \quad \Big\} \quad \log_2 1 + \log_2 1 = 0 + 0 = \underline{\underline{0}}$$

<u>Ex:</u> expand:

- $\log_5 7^4 = \boxed{4\log_5 7}$

- $\log_3(8^2 \cdot x^3) = \log_3 8^2 + \log_3 x^3$
  $= \boxed{2\log_3 8 + 3\log_3 x}$

- $\ln\left(\dfrac{2x}{3^3y^2}\right) = \ln(2x) - \ln(3^3y^2)$
  $= \ln 2 + \ln x - (\ln 3^3 + \ln y^2)$

  $= \ln 2 + \ln x - \ln 3^3 - \ln y^2$
  $= \boxed{\ln 2 + \ln x - 3\ln 3 - 2\ln y}$