12/05

Tuesday, December 5, 2017 4:21 PM

Online office hour on Wed. 12/6 at 7PM.

A review (from precalc) for Test 4 is available:

https://mathstat.fiu.edu/useful-information/math-resources/pre-calculus-algebra/review-chapter-4.pdf

Section 4.3

Properties of logs/exp

1) • $e^{\ln x} = x \Leftrightarrow b^{\log_b x} = x$
   • $\log_b b^x = x$

2) $\log_b(M \cdot N) = \log_b M + \log_b N$

3) $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

4) $\log_b(M^p) = p \cdot \log_b M$

5) (change of base prop)

$$\log_b M = \frac{\log_a M}{\log_a b}$$

Ex: Expand

$$\log_6\left(\frac{\sqrt[3]{x}}{36y^4}\right) = \log_6(x^{1/3}) - \log_6(6^2y^4)$$

$$= \tfrac{1}{3}\log_6 x - (\log_6 6^2 + \log_6 y^4)$$

$$= \tfrac{1}{3}\log_6 x - \log_6 6^2 - \log_6 y^4$$

$$= \boxed{\tfrac{1}{3}\log_6 x - 2 - 4\log_6 y}$$

- $\log_b(x^2\sqrt{y}) = \log_b(x^2y^{\frac{1}{2}})$

$$= \log_b x^2 + \log_b y^{\frac{1}{2}}$$

$$= \boxed{2\log_b x + \tfrac{1}{2}\log_b y}$$

Ex: Write as a single logarithm

- $\log_4 2 + \log_4 32 = \log_4(2\cdot 32) = \log_4 64$

$$= \log_4(4^3) = 3\log_4 4 = \boxed{3}$$

- $\log(4x-3) - \log x = \boxed{\log\left(\frac{4x-3}{x}\right)}$

- $\frac{1}{2}\log x + 4\log(x-1) = \log x^{1/2} + \log(x-1)^4$

$= \boxed{\log\left(\sqrt{x}\,(x-1)^4\right)}$

- $3\ln(x+7) - \ln x = \ln(x+7)^3 - \ln x$

$= \boxed{\ln\left(\frac{(x+7)^3}{x}\right)}$

- $4\log_b x - 2\log_b 6 - \frac{1}{2}\log_b y$

$= \underbrace{\log_b x^4 - \log_b 6^2 - \log_b \sqrt{y}}$

$= \log_b\left(\frac{x^4}{6^2}\right) - \log_b \sqrt{y}$

$= \boxed{\log_b\left(\frac{x^4}{6^2 y^{1/2}}\right)}$

$= \log_b x^4 - \left(\log_b 6^2 + \log_b y^{1/2}\right)$

$= \log_b x^4 - \log_b\left(6^2 \cdot y^{1/2}\right)$

$= \boxed{\log_b\left(\frac{x^4}{6^2 \cdot y^{1/2}}\right)}$

Evaluate:

$$\log_5 140 = \frac{\log 140}{\log 5} \left(= \frac{\ln 140}{\ln 5} \right) \qquad \log_5 140 = y \Updownarrow 5^y = 140$$

$$= \frac{2.146128}{0.69897} = 3.070415$$

Section 4.4

Solve: $2^x = \left(\frac{1}{2}\right)^{2x-1} \longrightarrow \left(\left(\frac{1}{2}\right)^{-1}\right)^x = \left(\frac{1}{2}\right)^{2x-1}$

$$\left(\frac{1}{2}\right)^{-x} = \left(\frac{1}{2}\right)^{2x-1}$$

$$2^x = \left(2^{-1}\right)^{2x-1}$$

$$-x = 2x - 1$$

$$\boxed{x = \frac{1}{3}}$$

$$2^x = 2^{-(2x-1)}$$

$$2^x = 2^{-2x+1} \longrightarrow x = -2x+1$$

$$3x = 1$$

$$\boxed{x = 1/3}$$

Solve: $2^x = 3^{2x}$ apply $\log(\cdot)$ to both sides

$$\log(2^x) = \log(3^{2x})$$

$$x \log 2 = 2x \log 3$$

$$-2x\log 3 \quad -2x\log 3$$

$$x\log 2 - 2x\log 3 = 0$$

factor x

$$\frac{x(\log 2 - 2\log 3)}{\log 2 - 2\log 3} = \frac{0}{\log 2 - 2\log 3}$$

$$x = \frac{0}{\log 2 - 2\log 3} = 0$$

Solve:

- $4^x = 15$

ln of both sides

$$\ln(4^x) = \ln 15$$

$$\frac{x \cancel{\ln 4}}{\cancel{\ln 4}} = \frac{\ln 15}{\ln 4}$$

$$\boxed{x = \frac{\ln 15}{\ln 4}} = \boxed{\log_4 15}$$

- $40e^{2x} - 3 = 237$

$$\frac{40e^{2x}}{40} = \frac{240}{40}$$

+3 +3

$$e^{2x} = 6$$

ln(··)

$$\ln e^{2x} = \ln 6$$

$$2x \ln e = \ln 6$$

$$\frac{2x}{2} = \frac{\ln 6}{2}$$

$$\boxed{x = \frac{\ln 6}{2}}$$

Solve: $\log_4 (x+3) = 2$

apply $4^{\square} = 4^{\square}$ on both sides

$$4^{\log_4 (x+3)} = 4^2$$

$$x+3 = 16$$

$$x = 13$$

$\rightarrow$ check:

$$\log_4(13+3) \overset{?}{=} 2$$

$\log_4(16) = 2$ ✓

- $\log_2 x + \log_2(x-7) = 3$

$\log_2(x(x-7)) = 3$

$\log_2(x^2 - 7x) = 3$

$2^{\square} = 2^{\square}$

$x^2 - 7x = 2^3$

$x^2 - 7x - 8 = 0$

$(x-8)(x+1) = 0$

$\underline{x = 8}$ $\qquad$ $\underline{x = -1}$

check

$\log_2 8 + \log_2(8-7) = 3$

$3 + \log_2(1) = 3$

$3 = 3$ ✓

check:

$\log_2(-1) + \log_2(-1-7) = 3$

DNE ✗

$$\boxed{\text{The only solution is } x = 8}$$

Solve: $e^{2x} - 4e^x + 3 = 0$

$(3(x-2)^2 + 2(x-2) + 1 = 0)$

$3u^2 + 2u + 1 = 0$

$(e^x)^2 - 4(e^x) + 3 = 0$

$3u^2+2u+1=0$

$$(e^x)^2-4(e^x)+3=0$$

$$\underline{u=e^x}$$

$$u^2-4u+3=0$$

$$(u-3)(u-1)=0$$

$u=3$ $\quad\quad$ $u=1$

$e^x=3$ $\quad$ $\ln\ldots=\ln\ldots$ $\quad\quad$ $e^x=1$

$\ln e^x=\ln 3$ $\quad\quad$ $\ln e^x=\ln 1$

$x\cdot\ln e=\ln 3$ $\quad\quad$ $x\cdot 1=0$

$\boxed{x=\ln 3}$ $\quad\quad$ $\boxed{x=0}$