12/06

Wednesday, December 6, 2017 7:02 PM

- $f(x) = x^2 - \frac{1}{x}, \quad g(x) = \sqrt{x-1} + 3$

Find

$$f(g(x)) = (f \circ g)(x) = f(\sqrt{x-1}+3)$$
$$= \boxed{(\sqrt{x-1}+3)^2 - \frac{1}{\sqrt{x-1}+3}}$$

$$g(f(x)) = g\left(x^2 - \frac{1}{x}\right) = \boxed{\sqrt{\left(x^2 - \frac{1}{x}\right) - 1} + 3}$$

$$f(x) = \frac{3}{x}, \quad g(x) = \frac{x}{x-1}$$

$D_f: x \neq 0 \qquad D_g: x \neq 1$

$$f(g(x)) = f\left(\frac{x}{x-1}\right) = \frac{3}{\frac{x}{x-1}} = \frac{3}{1} \cdot \frac{x-1}{x} = \boxed{\frac{3(x-1)}{x}}$$

Domain: $\{x \mid x \neq 1, 0\} = (-\infty, 0) \cup (0,1) \cup (1, \infty)$

$$g(f(x)) = g\left(\frac{3}{x}\right) = \frac{\frac{3}{x}}{\frac{3}{x} - 1} = \frac{\frac{3}{x}}{\frac{3-x}{x}} = \frac{3}{\cancel{x}} \cdot \frac{\cancel{x}}{3-x} = \boxed{\frac{3}{3-x}}$$

Domain: $\{x \mid x \neq 0, 3\}$

$f^{-1}(x)$ is an inverse of $f(x)$ if $f(f^{-1}(x)) = x$

and $f^{-1}(f(x)) = x$

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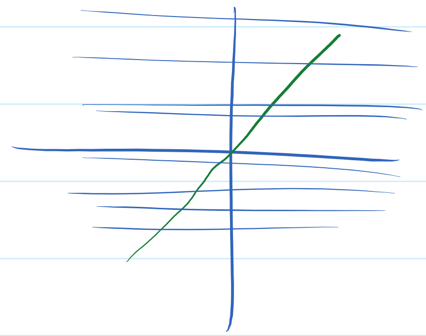
and $f^{-1}(f(x)) = x$

Find the inverse of $f(x) = \frac{3x}{2}$



$$y = \frac{3x}{2}$$

$$x = \frac{3y}{2}$$

solve for y...

$$\frac{2x}{3} = \frac{3y}{3}$$

$$\boxed{\frac{2x}{3} = y} \rightarrow \boxed{f^{-1}(x) = \frac{2x}{3}}$$

Find the inverse of: $y = f(x) = \frac{x-2}{x+1}$

$$x = \frac{y-2}{y+1}$$

$$x(y+1) = y-2$$

$$xy + x = y - 2$$

$$xy - y = -2 - x$$

$$\frac{y(x-1)}{x-1} = \frac{-2-x}{x-1}$$

$$y = \boxed{\frac{-2-x}{x-1}} \cdot \frac{-1}{-1} = \boxed{\frac{2+x}{1-x}}$$

$y = \boxed{\dfrac{\;}{x-1}} \cdot \dfrac{-1}{-1} = \boxed{\dfrac{\;}{1-x}}$

$f(x) = \dfrac{x-2}{x+1}$ $\qquad$ $\boxed{f^{-1}(x) = \dfrac{2+x}{1-x}}$

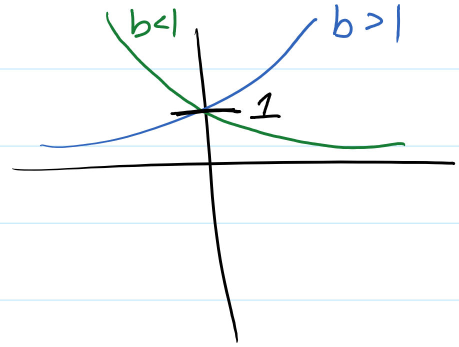
Dom: $\{x \mid x \neq -1\}$ $\qquad$ Dom: $\{x \mid x \neq 1\}$

Range: $\{x \mid x \neq 1\}$ $\qquad$ Range: $\{x \mid x \neq -1\}$

Exponential functions: $y = b^x$, $b > 0$, $b \neq 1$

Dom: $(-\infty, \infty)$

Range: $(0, \infty)$

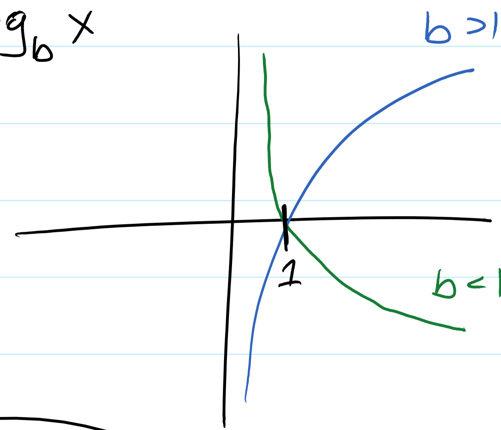


The inverse of $y = b^x \to x = b^y$

$y = \log_b x$

Dom: $(0, \infty)$

Range: $(-\infty, \infty)$

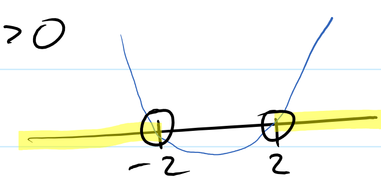


Find domain of:

- $\log_{\frac{1}{3}}(x^2 - 4) \longrightarrow x^2 - 4 > 0$

$(x-2)(x+2) > 0$

$x = 2, -2$



$$\boxed{\text{Dom: } (-\infty,-2)\cup(2,\infty)}$$

- $\log_4(x^3-2x^2+x) \to x^3-2x^2+x>0$

$$x(x^2-2x+1)>0$$

$\underline{x=0}$

$$x=\frac{2\pm\sqrt{4-4\cdot 1}}{2}=\frac{2\pm\sqrt{0}}{2}$$

$$=\frac{2}{2}=\underline{1}$$

0 1

	$(-\infty,0)$	$(0,1)$	$(1,\infty)$
pt	-1	0.5	2
x	$-$	$+$	$+$
$(x-1)^2$	$+$	$+$	$+$
$f(x)$	$-$	$+$	$+$

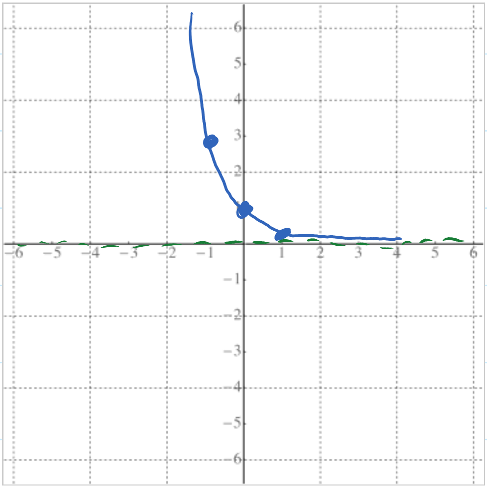
$$\boxed{\text{Dom: } (0,1)\cup(1,\infty)}$$

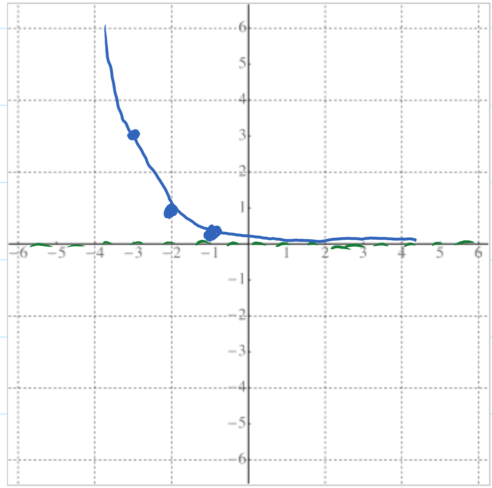
$\cancel{=(0,\infty)}$?

Graph using transformations:

$\underline{\left(\frac{1}{3}\right)^{-x+2}}$

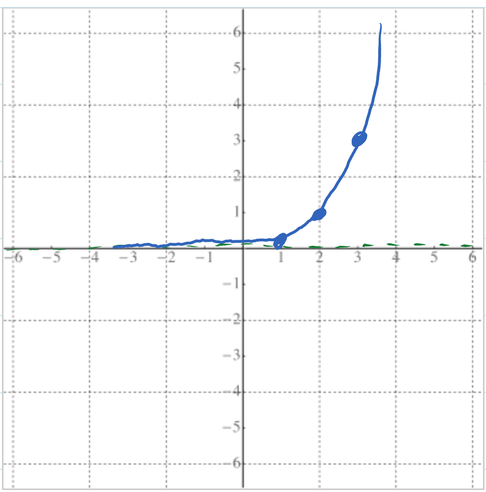
base: $\left(\frac{1}{3}\right)^{x} \xrightarrow[\text{by } 2]{\text{hor. shift left}} y=\left(\frac{1}{3}\right)^{x+2}$





reflection about y-axis

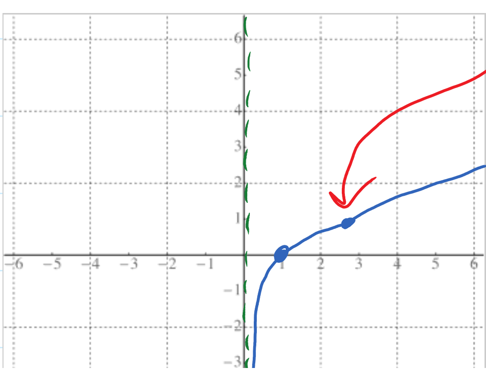
$$y = \left(\frac{1}{3}\right)^{-x+2}$$



Graph: $2\ln\left(\frac{x}{3}+1\right)$

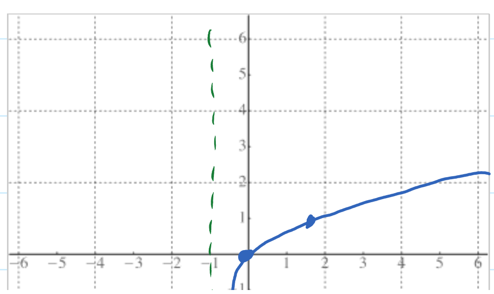
base: $y = \ln(x) = \log_e(x)$ $\leftarrow$ $\log_b b = 1$ $\rightarrow$ $\log_e e = \ln e = 1$

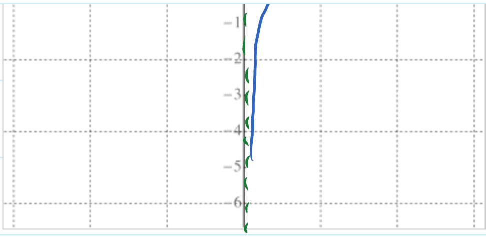
$\boxed{(e,1)}$

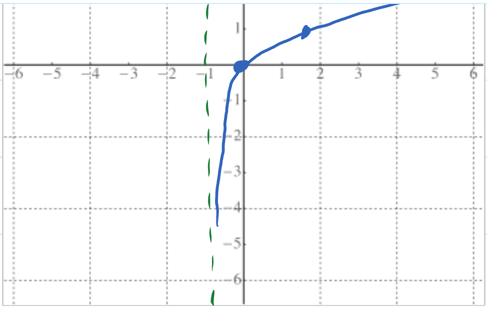


$\longrightarrow$ hor. shift left by 1

$$y = \ln(x+1)$$

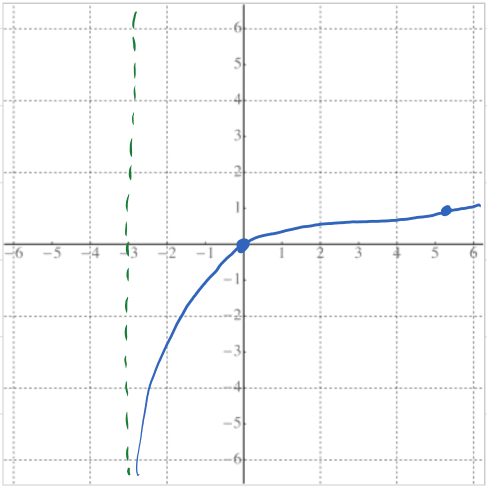






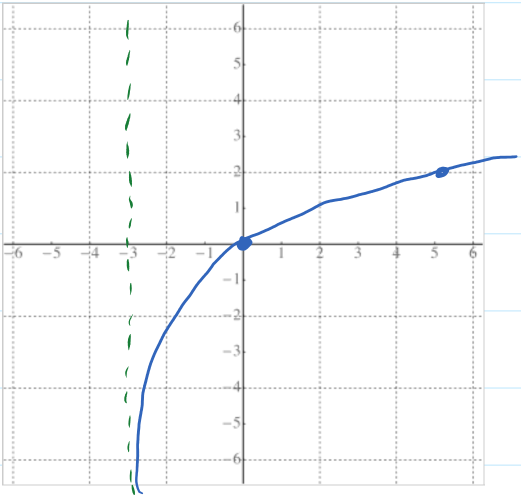
hor. stretch by 3

$y = \ln\left(\frac{x}{3}+1\right)$



vert. stretch by 2

$y = 2\ln\left(\frac{x}{3}+1\right)$



Solve: $\ln(x-6) + \ln(x+1) = \ln(x-15)$

$$e^{\ln[(x-6)\cdot(x+1)]} = e^{\ln(x-15)}$$

$b^{\log_b x} = x$

$$(x-6)(x+1) = x-15$$
$$x^2 - 5x - 6 = x - 15$$
$$x^2 - 6x + 9 = 0$$
$$(x-3)(x-3) = 0$$
$$x = 3 \longrightarrow \text{check: } x = 3$$

$(x-3)(x-3)=0$

$\underline{x=3} \longrightarrow$ check: $x=3$

$\ln(3-6)+\ln(3+1)=\ln(3-15)$

$\ln(-3)+\ln(4)=\ln(-12)$

x=3 is not a solution

Solve: $\log_2(3x-2)-\log_2(x-5)=4$

$$\log_2\left(\frac{3x-2}{x-5}\right)=4 \Longleftrightarrow \frac{3x-2}{x-5}=2^4$$

$$\frac{3x-2}{x-5}=16$$

$3x-2=(x-5)16$

$3x-2=16x-80$

$3x-16x=+2-80$

$-13x=-78$

$x=\frac{-78}{-13}=\underline{6} \longrightarrow$ check:

$\log_2(3\cdot6-2)-\log_2(6-5)=4$

$\log_2(16)-\log_2 1=4$

$4-0=4$ ✓

$$\boxed{x=6}$$

Solve: $3\cdot 2^{2x-1}+5=14$

$\qquad -5 \quad -5$

$3\cdot 2^{2x-1}=9$

$$\frac{3\cdot 2^{2x-1}}{3} = \frac{9}{3}$$

$$2^{2x-1} = 3 \qquad |\log_2 \cdots$$

$$\log_2 2^{2x-1} = \log_2 3$$

$$(2x-1)\underbrace{\log_2 2}_{=1} = \log_2 3$$

$$2x-1 = \log_2 3$$

$$\frac{2x}{2} = \frac{1+\log_2 3}{2}$$

$$x = \boxed{\frac{1+\log_2 3}{2}}$$

$\log_b M^p = p\cdot\log_b M$

$\log_b(M\cdot N) = \log_b M + \log_b N$

$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

$\log_b M = \dfrac{\log_a M}{\log_a b}$

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expand $\log\left(\dfrac{2x^3y}{3z}\right)$

write as one log : $3\log_a x - 2\log_a y + \log_a z$

$$\underbrace{\ln\left(\frac{x}{x-8}\right) + \ln\left(\frac{x+8}{x}\right)} - \ln(x^2-64)$$

$$= \ln\left(\frac{\cancel{x}}{x-8}\cdot\frac{x+8}{\cancel{x}}\right) - \ln(x^2-64)$$

$$= \ln\left(\frac{x+8}{x-8}\right) - \ln(x^2-64)$$

$$= \ln\left(\frac{\frac{x+8}{x-8}}{x^2-64}\right) = \ln\left(\frac{x+8}{x-8}\cdot\frac{1}{x^2-64}\right)$$

$$= \ln\left(\frac{\cancel{x+8}}{x-8}\cdot\frac{1}{(\cancel{x+8})(x-8)}\right) = \boxed{\ln\left(\frac{1}{(x-8)^2}\right)}$$

$$= \ln\left((x-8)^{-2}\right) = \boxed{-2\ln(x-8)}$$