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Tuesday, August 22, 2017 5:18 PM

P.4 Polynomials

$3x-31,\ x^2-3x+1,\ 4+7,\ \sqrt{x}-1,\ \frac{1}{x}$

Polynomials — Not polynomials

$2x-5$, Pol; x^3-2x+1, Pol; $\frac{1}{2}x-2$ (or $0.5\cdot x-2$), Pol; $3x$, Pol; $\sqrt{2}$, Pol; $\frac{1}{3}x-x^2$, Pol

$\sqrt{2}x-2$, Pol; $x^4-\sqrt{x}$, Not P.; $0.2x+1$, Pol.; $\sqrt{2x}-2$, Not Pol

"Def": A polynomial in x is a expression of the form:

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0,$$

$n=2$: $a_2 x^2 + a_1 x + a_0$

$n=3$: $a_3 x^3 + a_2 x^2 + a_1 x + a_0$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers, $a_n \neq 0$. and n is non negative.

real number: $2, 21, 7, 0.2, \sqrt{2}, \sqrt{3}, \pi, 123456, \frac{1}{3}, \frac{22}{3}, -2, -\sqrt{2}$

$i = \sqrt{-1}$ ← not a real number!

$\sqrt{-16}$ ← not a real number!

$6x^4 - 4x^2 + 3$

Degree of a polynomial is the highest exponent

Polyn.	degr
$6x^4 - x^2 + 2$	4
$x^3 - 21x^5 + 3$	5
$-9x^2 - 6x + 5$	2
$\sqrt{3}x + x^{11}$	11

$(-9x^3 + 7x^2 - 5x + 3) + (13x^3 + 2x^2 - 8x - 6)$

$-9x^3 + 13x^3 + 7x^2 + 2x^2 - 5x - 8x + 3 - 6$

$4x^3 + 9x^2 - 13x - 3$

$$(-9x^3+7x^2-5x+3)-(13x^3+2x^2-8x-6)$$
$$=-9x^3+7x^2-5x+3-13x^3-2x^2+8x+6$$
$$=\boxed{-22x^3+5x^2+3x+9}$$

$$(-8x^6)(5x^3)=-8\cdot 5\, x^6\cdot x^3=-40\,x^{6+3}=\boxed{-40x^9}$$

$$(2x+3)(x^2+4x+5)=2x\cdot x^2+2x\cdot 4x+2x\cdot 5+3\cdot x^2$$
$$+3\cdot 4x+3\cdot 5$$
$$=2x^3+8x^2+10x+3x^2+12x+15$$
$$=\boxed{2x^3+11x^2+22x+15}$$

$$(3x+4)(5x-3)=15x^2+20x-9x-12=\boxed{15x^2+11x-12}$$

$$(2y-1)(2y+1)=4y^2+2y-2y-1=4y^2-1=(2y)^2-1^2$$

$$(7x+8)(7x-8)=49x^2-56x+56x-64=49x^2-64$$
$$=(7x)^2-8^2$$

$$\boxed{(a+b)(a-b)=a^2-b^2}$$

$(x+10)^2 = (x+10)(x+10) = x^2 + 10x + 10x + 100$

$= x^2 + 20x + 100$

$(a+b)^2 = a^2 + 2ab + b^2$

$2x^2 \neq (2x)^2$

$\| \quad$

$4x^2$

$(2x+4)^2 = (2x)^2 + 2 \cdot (2x)(4) + 4^2$

$a \quad b$

$= \boxed{4x^2 + 16x + 16}$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a-b)(a-b) = a^2 + a(-b) + (-b)a + (-b)(-b)$

$= a^2 - ab - ba + b^2 = a^2 - ab - ab + b^2$

$= a^2 - 2ab + b^2$

$a \quad b$

$(5y-6)^2 = (5y)^2 - 2(5y) \cdot 6 + 6^2 = 5^2 \cdot y^2 - 60y + 36$

$= \boxed{25y^2 - 60y + 36}$

$-2 \cdot (5y) \cdot 6$

$-10y \cdot 6$

$-60y$

$(7x-3)^2 = (7x)^2 - 2 \cdot 7x \cdot 3 + 9$

$49x^2 - 42x + 9$

$$(a+b)^3 = (a+b)^2 \cdot (a+b) = (a^2+2ab+b^2)(a+b)$$
$$= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3$$

$$= \boxed{a^3 + 3a^2b + 3ab^2 + b^3}$$

$$(a-b)^3 = \boxed{a^3 - 3a^2b + 3ab^2 - b^3}$$

$$(2x-1)^3 = (2x)^3 - 3\cdot(2x)^2\cdot 1 + 3\cdot 2x\cdot(1)^2 - 1^3$$
$$= 8x^3 - 3\cdot 4x^2 + 6x - 1 = \boxed{8x^3 - 12x^2 + 6x - 1}$$

(a = 2x, b = 1)

Section P.5 Factoring

$$18x^3 + 27x^2 = 9(2x^3 + 3x^2) = 9x^2(2x+3)$$

$$x^2(x+3) + 5(x+3) = (x+3)(x^2+5)$$

$a^2 - b^2 = (a-b)(a+b)$

$$x^4(x-2) - 81(x-2) = (x-2)(x^4 - 81)$$
$$= (x-2)(x^4 - 9^2)$$

$$= (x-2)((x^2)^2 - 9^2)$$

$$= (x-2)\left((x^2)^2 - 9^2\right)$$

$$= (x-2)(x^2-9)(x^2+9)$$

$$x^2 - 3^2$$

$$= \boxed{(x-2)(x-3)(x+3)(x^2+9)}$$