

Section 1.5

The general form of a quadratic equation is

$$ax^2 + bx + c = 0,$$

where a, b and c are real numbers, with $a \neq 0$.

Factoring

Solve:

$$4x^2 - 2x = 0$$

$$2x(2x - 1) = 0$$

$$\frac{2x}{2} = \frac{0}{2}$$

$$\boxed{x = 0}$$

$$2x - 1 = 0$$

$$+1 = +1$$

$$\frac{2x}{2} = \frac{1}{2}$$

$$\boxed{x = \frac{1}{2}}$$

The solution set: $\{0, \frac{1}{2}\}$

Solve: $2x^2 + 7x = 4$

$$\begin{array}{r} -4 \quad -4 \end{array}$$

$$\cancel{x(2x+7)=4}$$

$$2x^2 + 7x - 4 = 0$$

$2 \cdot (-4) = -8$
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Find A, C so that

$$\left. \begin{array}{l} A \cdot C = -8 \\ A + C = 7 \end{array} \right\} \begin{array}{l} A = 8 \\ C = -1 \end{array}$$

Rewrite $7x$ as $8x - 1x$

$$2x^2 + 8x - 1x - 4 = 0$$

$$2x(x+4) - (x+4) = 0$$

$$(x+4)(2x-1) = 0$$

$$x+4=0$$

-4 -4

$$\boxed{x = -4}$$

$$2x-1=0$$

+1 +1

$$\frac{2x}{2} = \frac{1}{2}$$

$$\boxed{x = 1/2}$$

Solve : $2x^2 + x = 1$

$$2x^2 + x - 1 = 0$$

$$2x^2 + x - 1 = 0$$

$$\left. \begin{array}{l} A \cdot C = -2 \\ A + C = 1 \end{array} \right\} \begin{array}{l} A = -1 \\ C = 2 \end{array}$$

$$2x^2 - x + 2x - 1 = 0$$

$$x(2x-1) + (2x-1) = 0$$

$$(2x-1)(x+1) = 0$$

$$2x-1=0$$

$$\boxed{x = \frac{1}{2}}$$

$$x+1=0$$

$$\boxed{x = -1}$$

Square root property:

$$3x^2 - 15 = 0$$

$$+15 \quad +15$$

$$\frac{3x^2}{3} = \frac{15}{3}$$

$$x^2 = 5 \rightarrow \boxed{x = \sqrt{5}}, \boxed{x = -\sqrt{5}}$$

$$9x^2 + 25 = 0$$

$$-25 \quad -25$$

$$\frac{9x^2}{9} = \frac{-25}{9}$$

9 9

$$x^2 = -\frac{25}{9} \rightarrow x = \sqrt{-\frac{25}{9}} \text{ or } x = -\sqrt{-\frac{25}{9}}$$

$$x = i\sqrt{\frac{25}{9}} \quad x = -i\sqrt{\frac{25}{9}}$$

$$x = i\frac{\sqrt{25}}{\sqrt{9}} \quad x = -i\frac{\sqrt{25}}{\sqrt{9}}$$

$$x = \boxed{i\frac{5}{3}} \quad x = \boxed{-i\frac{5}{3}}$$

Solve $(x-2)^2 = 6$ $y^2 = 6 \rightarrow y = \sqrt{6}$ $y = -\sqrt{6}$

$$x-2 = \sqrt{6}$$

+2 +2

$$x = \boxed{2 + \sqrt{6}}$$

$$x-2 = -\sqrt{6}$$

+2 +2

$$x = \boxed{2 - \sqrt{6}}$$

Complete the square:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$x^2 + 8x = \left(x + \frac{8}{2}\right)^2 - \frac{8^2}{4} = (x+4)^2 - \frac{64}{4} = (x+4)^2 - 16$$

$$b = 8$$

$$\parallel$$
$$x^2 + 2x \cdot 4 + 4^2 - 16$$

$$x^2 + 2x \cdot 4 + 4^2 - 16$$
$$x^2 + 8x + 16 - 16$$

Compl. the square

$$\bullet x^2 - 7x = \left(x + \frac{-7}{2}\right)^2 - \frac{(-7)^2}{4} = \boxed{\left(x - \frac{7}{2}\right)^2 - \frac{49}{4}}$$

$b = -7$

$$\bullet x^2 + \frac{3}{5}x = \left(x + \frac{3}{10}\right)^2 - \frac{9}{100}$$

$$b = \frac{3}{5}$$

$$\frac{b}{2} = \frac{\frac{3}{5}}{2} = \frac{\frac{3}{5}}{\frac{2}{1}} = \frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10}$$

$$\frac{b^2}{4} = \frac{\left(\frac{3}{5}\right)^2}{4} = \frac{\frac{9}{25}}{4} = \frac{9}{100}$$

Solve by completing the square

$$\underline{x^2 - 6x} + 4 = 0$$

$$b = -6$$

$$\underline{\left(x - \frac{6}{2}\right)^2 - \frac{6^2}{4}} + 4 = 0$$

$$(x-3)^2 - \frac{36}{4} + 4 = 0$$

$$(x-3)^2 - 9 + 4 = 0$$

$$(x-3)^2 - 5 = 0$$

$$(x-3)^2 = 5 \rightarrow x-3 = \sqrt{5}$$

$$+3 \quad +3$$

$$x = \boxed{3 + \sqrt{5}}$$

$$x-3 = -\sqrt{5}$$

$$+3 \quad +3$$

$$x = \boxed{3 - \sqrt{5}}$$

Solve: $9x^2 - 6x - 4 = 0$

To complete the square
divide eq. by 9.

$$\frac{9x^2}{9} - \frac{6x}{9} - \frac{4}{9} = \frac{0}{9}$$

$$x^2 - \frac{2x}{3} - \frac{4}{9} = 0$$

$$\boxed{x^2 - \frac{2}{3}x} - \frac{4}{9} = 0$$

$$\boxed{\left(x - \frac{1}{3}\right)^2} - \frac{1}{9} - \frac{4}{9} = 0$$

$$\left(x - \frac{1}{3}\right)^2 - \frac{1+4}{9} = 0$$

$$b = -\frac{2}{3}$$

$$\frac{b}{2} = -\frac{2}{6} = \left(-\frac{1}{3}\right)$$

$$\frac{b^2}{4} = \frac{\left(-\frac{2}{3}\right)^2}{4} = \frac{4}{9}$$

$$= \frac{4}{9} = \left(\frac{2}{3}\right)$$

$$\left(x - \frac{1}{3}\right)^2 - \frac{1-4}{9} = 0 \qquad = \frac{4}{4 \cdot 9} = \left(\frac{1}{9}\right)$$

$$\left(x - \frac{1}{3}\right)^2 - \frac{5}{9} = 0$$

$$\left(x - \frac{1}{3}\right)^2 = \frac{5}{9} \rightarrow x - \frac{1}{3} = \sqrt{\frac{5}{9}} \qquad x - \frac{1}{3} = -\sqrt{\frac{5}{9}}$$

$$x - \frac{1}{3} = \frac{\sqrt{5}}{3}$$

$$x - \frac{1}{3} = -\frac{\sqrt{5}}{3}$$

$$x = \frac{\sqrt{5}}{3} + \frac{1}{3}$$

$$x = -\frac{\sqrt{5}}{3} + \frac{1}{3}$$

$$x = \frac{1 + \sqrt{5}}{3}$$

$$x = \frac{1 - \sqrt{5}}{3}$$

Use the quadratic formula:

$$\textcircled{9}x^2 - \textcircled{6}x - \textcircled{4} = 0$$

$$a \quad b \quad c$$

$$a = 9$$

$$b = -6$$

$$c = -4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 9 \cdot (-4)}}{2 \cdot 9}$$

$$= \frac{6 \pm \sqrt{36 + 36 \cdot 4}}{18}$$

$$\begin{array}{r} 36 \\ + 4 \\ \hline 144 \end{array}$$

$$= \frac{6 \pm \sqrt{36 + 36 \cdot 4}}{18}$$

$$\begin{array}{r} -7 \\ 144 \\ +36 \\ \hline 180 \end{array}$$

$$= \frac{6 \pm \sqrt{180}}{18} = \frac{6 \pm \sqrt{5 \cdot 36}}{18}$$

$$= \frac{6 \pm \sqrt{36 \cdot 5}}{18} = \frac{6 \pm 6\sqrt{5}}{18} = \frac{2(1 \pm \sqrt{5})}{3} = \boxed{\frac{1 \pm \sqrt{5}}{3}}$$

Solve: $3x^2 - 2x + 4 = 0$

$$\begin{array}{l} a = 3 \\ b = -2 \\ c = 4 \end{array}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 3 \cdot 4}}{2 \cdot 3}$$

$$x = \frac{2 \pm \sqrt{4 - 12 \cdot 4}}{6} = \frac{2 \pm \sqrt{4 - 48}}{6} = \frac{2 \pm \sqrt{-44}}{6}$$

$$= \frac{2 \pm i\sqrt{44}}{6} = \frac{2 \pm i\sqrt{4 \cdot 11}}{6} = \frac{2 \pm i \cdot 2\sqrt{11}}{6}$$

$$= \boxed{\frac{1 \pm i\sqrt{11}}{3}}$$