1 Quadratic equations

Write down the quadratic formula to solve the following equation: $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Can we use a part of this formula to find how many solutions a quadratic equation has? Explain.

yes:
- $b^2 - 4ac > 0 \Rightarrow$ 2 real solutions
- $b^2 - 4ac = 0 \Rightarrow$ 1 real solution
- $b^2 - 4ac < 0 \Rightarrow$ 2 complex solutions

Exercise 1.1. Solve for x, including any complex solutions.

1. $3x^2 = 27$

$x^2 = 9$

$x = \pm\sqrt{9} = \boxed{\pm 3}$

2. $(x+2)^2 = -7$

$x + 2 = \pm\sqrt{-7} = \pm i\sqrt{7}$

$x = \boxed{-2 \pm i\sqrt{7}}$

3. $(2x+3)(x+4) = 1$

$2x^2 + 8x + 3x + 12 = 1$

$2x^2 + 11x + 11 = 0$

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \cdot 2 \cdot 11}}{2 \cdot 2} = \frac{-11 \pm \sqrt{121 - 88}}{4}$$

$$= \boxed{\frac{-11 \pm \sqrt{33}}{4}}$$

4. $2x^2 - 7x = 0$

$x(2x-7)=0$

$x=0$

$2x-7=0$

$x=7/2$

2 Radical equations

Exercise 2.1. Solve for x.

1. $\left(\sqrt{3x+18}\right)^2 = x^2$

$3x+18=x^2$

$0=x^2-3x-18$

$0=(x-6)(x+3)$

$x-6=0$ $\quad x+3=0$

$x=6$ $\quad x=-3$

$x=6$

$\sqrt{3\cdot 6+18}=6$

$\sqrt{36}=6$

OK

$x=-3$

$\sqrt{3(-3)+18}=-3$

$\sqrt{9}=-3$

NO

2. $\sqrt{x+2}+\sqrt{3x+7}=1$

(Can we just square each side or do we have to do something before?)

$\left(\sqrt{x+2}\right)^2=\left(1-\sqrt{3x+7}\right)^2$

$x+2=1-2\cdot 1\cdot\sqrt{3x+7}+\left(\sqrt{3x+7}\right)^2$

$x+2=1-2\sqrt{3x+7}+3x+7$

$x+2-1-3x-7=-2\sqrt{3x+7}$

$(-2x-6)^2=\left(-2\sqrt{3x+7}\right)^2$

$(-2x)^2+2\cdot(-2x)\cdot(-6)+(-6)^2=(-2)^2\left(\sqrt{3x+7}\right)^2$

$4x^2+24x+36=4(3x+7)$

$4x^2+24x+36-12x-28=0$

$4x^2+12x+8=0$

$x^2+3x+2=0$

$(x+2)(x+1)=0$

$x=-2$ $\quad x=-1$

$x=-2$

$\sqrt{-2+2}+\sqrt{-6+7}=1$

$0+\sqrt{1}=1$

yes

$x=-1$

$\sqrt{-1+2}+\sqrt{-3+7}=1$

$\sqrt{1}+\sqrt{4}=1$

$1+2=1$

no

3 Lines

Write down the formula for the **distance** between the points (x_1, y_1) and (x_2, y_2).

$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ or $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Write down the formula for the **midpoint** between the points (x_1, y_1) and (x_2, y_2).

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Exercise 3.1. Find the distance between the points $(0, 1)$ and $(3, 5)$. What is the midpoint between the points? Simplify your answer.

distance: $\sqrt{(0-3)^2+(1-5)^2} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$

midpoint: $\left(\frac{0+3}{2}, \frac{1+5}{2}\right) = \boxed{\left(\frac{3}{2}, 3\right)}$

How can you algebraically find out if two lines are perpendicular or parallel?

$y = mx+b$ perpendicular: $m_1 = -\frac{1}{m_2}$ or $m_1 \cdot m_2 = -1$

parallel: $m_1 = m_2$

Exercise 3.2. Find the slope of the line given by the equation: $3x + y = 4$. Find the slope-intercept form of a line that is perpendicular to this line and passes through the point (9,2).

$3x+y=4$ slope is $\boxed{-3}$

$y = -3x+4$

perpendicular: slope $\frac{1}{3}$

$y-2 = \frac{1}{3}(x-9)$

$y = \frac{1}{3}x-3+2$

$\boxed{y = \frac{1}{3}x-1}$

4 Circles

What is the difference between the general form and the standard form of the equation of a circle?

General form: $x^2+y^2+ax+by+c=0$

standard form: $(x-h)^2+(y-k)^2=r^2$

Exercise 4.1. Write the standard form of the equation of the circle with the given center and radius.

1. Center (0,0), r=3

$x^2+y^2=3^2$ or $x^2+y^2=9$

2. Center (0,2), r=5

$x^2+(y-2)^2=5^2$ or $x^2+(y-2)^2=25$

Exercise 4.2. Convert the general form of a circle's equation to standard form and vice versa. Find the center and radius of the circle.

1. $(x+3)^2+(y-1)^2=4$

center: $(-3,1)$
radius: 2

$x^2+6x+9+y^2-2y+1=4$

$x^2+y^2+6x-2y+6=0$

2. $x^2+y^2+8x-2y-8=0$

$x^2+8x+4^2+y^2-2y+(-1)^2=8+4^2+(-1)^2$

$(x+4)^2+(y-1)^2=8+16+1$

$(x+4)^2+(y-1)^2=25$

center $=(-4,1)$
radius $=5$