

MAC 1140, Fall 2017

Exam #1

September 25, 2017

Name Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who uses a cell phone during the examination or if one is found within hands reach.
- Calculators are not allowed on this exam.
- The exam consist of two parts. Part I contains four multiple choice questions worth 5 points each. Part II contains 8 open ended questions worth 10 points each if not stated otherwise.

Part I

Choose your answer from five available choices. No partial credit will be given for wrong answers.

1. What is $(f \circ g)(x) = f(g(x))$ if $f(x) = \sqrt{3x-2}$ and $g(x) = \frac{2}{x+4}$

(a) $\frac{2}{\sqrt{3x-2}+4}$

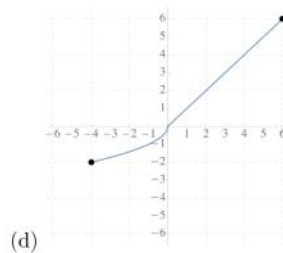
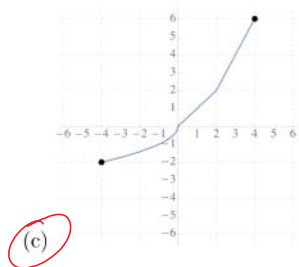
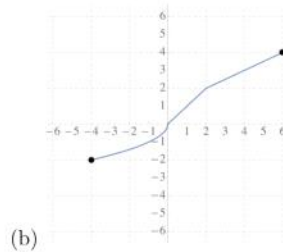
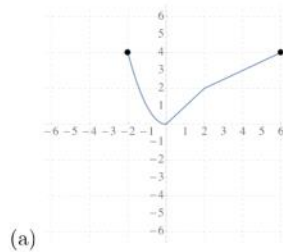
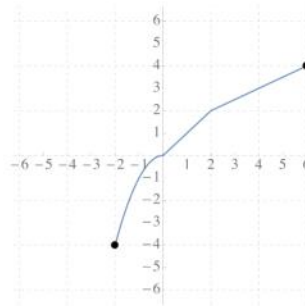
(b) $\frac{2\sqrt{3x-2}}{x+4}$

(c) $\frac{2}{\sqrt{3x-2}+4}$

(d) $\sqrt{3\frac{2}{x+4}-2}$

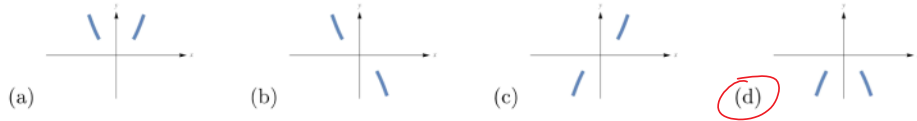
(e) None of the above

2. The graph of a one-to-one function f is given below. Which of the following is the graph of its inverse?



(e) None of the above.

3. Which of the following illustrates the end behavior of $f(x) = -2x^4 + 3x^2 - x + 7$



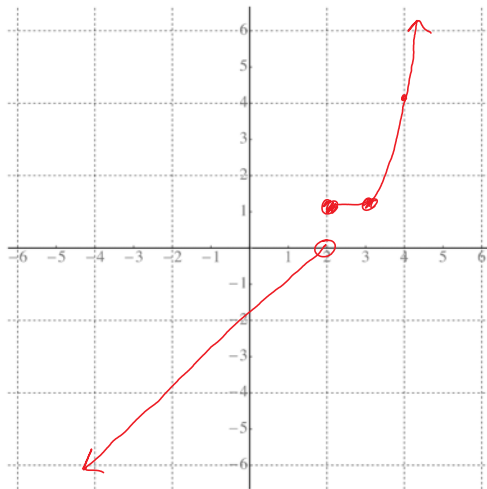
4. Find the domain of $f(x) = \frac{3-x}{\sqrt{x+2}}$

- (a) $(-\infty, -2)$
- (b) $(-2, 3) \cup (3, \infty)$
- (c) $[-2, \infty)$
- (d) $(-2, \infty)$**
- (e) None of the above.

$$\begin{array}{l}
 x+2 \geq 0 \quad \text{and} \quad \sqrt{x+2} \neq 0 \\
 x \geq -2 \quad \quad \quad x+2 \neq 0 \\
 \quad \quad \quad \quad \quad \quad \quad \quad x \neq -2 \\
 \hline
 x > -2
 \end{array}$$

Part II

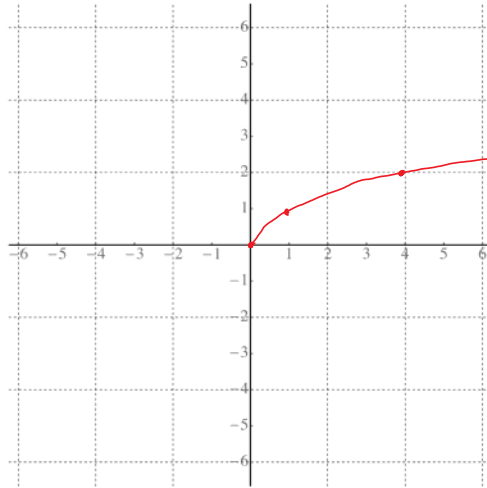
5. Graph the function $f(x) = \begin{cases} x-2 & , x < 2 \\ 1 & , 2 \leq x \leq 3 \\ (x-2)^2 & , x > 3 \end{cases}$



6. Graph $y = 2\sqrt{-x+1}$ using transformations. Start with the graph of a basic function – **plot accurately as least three points** and use them to perform transformations. Do one transformation at a time. Name the transformation and write the equation of the resulting function.

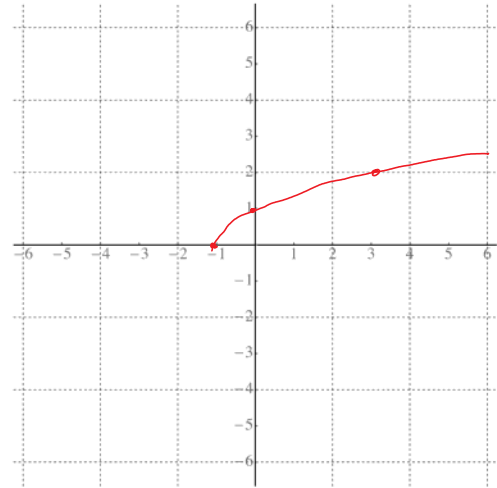
(i) Basic function:

$$y = \sqrt{x}$$



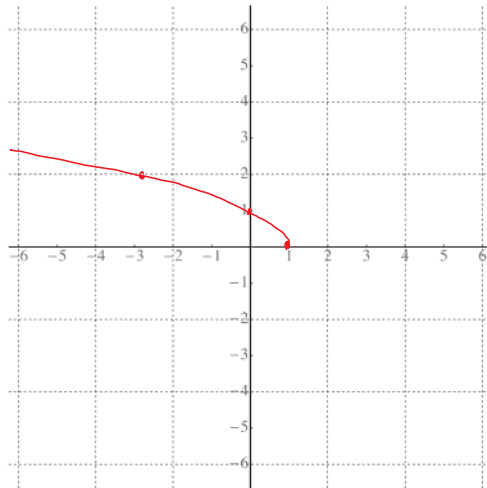
(ii) transformation: *hor. shift to left by one unit*

$$y = \sqrt{x+1}$$



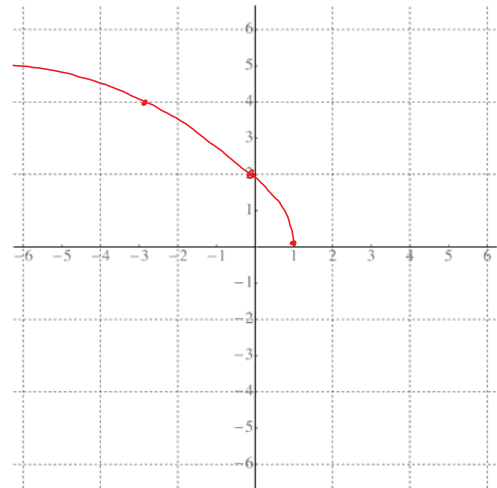
(iii) transformation: *reflection about y-axis*

$$y = \sqrt{-x+1}$$



(iv) transformation: *vertical stretch by factor of 2*

$$y = 2\sqrt{-x+1}$$



7. Find the difference quotient for $f(x) = -5x^2 + 3$.

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{-5(x+h)^2 + 3 - (-5x^2 + 3)}{h} \\ &= \frac{-5(x^2 + 2xh + h^2) + 3 + 5x^2 - 3}{h} = \frac{-5x^2 - 10xh - 5h^2 + 5x^2}{h} \\ &= \frac{-10x - 5h}{h} = \boxed{-10x - 5h} \end{aligned}$$

8. (20 points) Let $f(x) = 2 + 3\sqrt{1-x}$

(a) Find the domain of f .

$$\begin{aligned} 1-x &\geq 0 \\ 1 &\geq x \end{aligned} \quad \boxed{(-\infty, 1]}$$

(b) Find the inverse of f .

$$\begin{aligned} x &= 2 + 3\sqrt{1-y} \\ x-2 &= 3\sqrt{1-y} \\ \frac{x-2}{3} &= \sqrt{1-y} \\ \left(\frac{x-2}{3}\right)^2 &= 1-y \end{aligned}$$

$$\begin{aligned} -y &= \left(\frac{x-2}{3}\right)^2 - 1 \\ \boxed{y} &= 1 - \left(\frac{x-2}{3}\right)^2 \\ \text{Domain: } &[2, \infty) \end{aligned}$$

9. Find the zeros and their multiplicities and sketch the graph of the following polynomial.

$$y = -3(x^2 + 2)(x - 3)^2(x + 6)^3$$

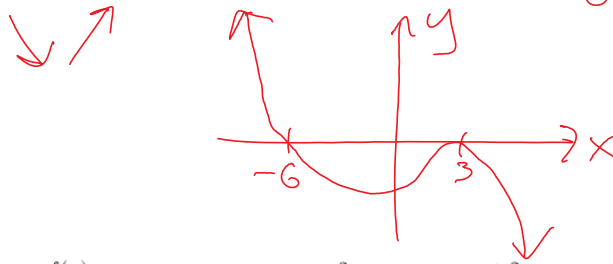
$$\begin{aligned} x^2 + 2 &= 0 \\ x^2 &= -2 \\ \text{No solution} \end{aligned}$$

$$\begin{aligned} x - 3 &= 0 \\ |x = 3| \end{aligned}$$

$$\begin{aligned} x + 6 &= 0 \\ |x = -6| \end{aligned}$$

degree: 7
leading coeff: -3

zero	multiplicity
3	2
-6	3



10. Find $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ and its domain if $f(x) = \frac{2}{x}$ and $g(x) = \frac{x+2}{1-x}$

$$\frac{f(x)}{g(x)} = \frac{\frac{2}{x}}{\frac{x+2}{1-x}} = \frac{2}{x} \cdot \frac{1-x}{x+2} = \boxed{\frac{2(1-x)}{x(x+2)}}$$

$$x \neq 1, 0, -2$$

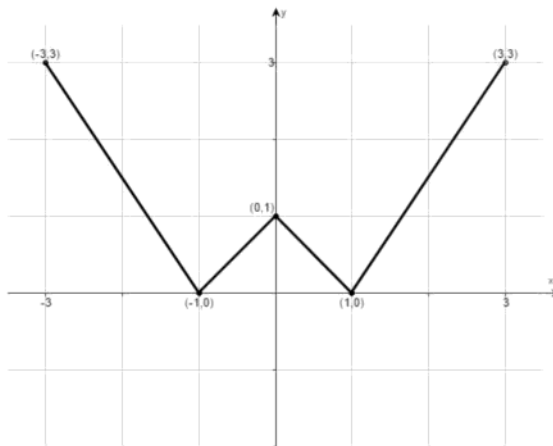
$$\text{Domain: } (-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, \infty)$$

11. Find two functions $f(x)$ and $g(x)$ (neither of them identity) so that $h(x) = f(g(x))$, where

$$h(x) = \frac{3}{2\sqrt{x+1}}$$

$$\left. \begin{aligned} g(x) &= \sqrt{x+1} \\ f(x) &= \frac{3}{2x} \end{aligned} \right\} \begin{aligned} g(x) &= x+1 \\ f(x) &= \frac{3}{2\sqrt{x}} \end{aligned}$$

12. Using the given graph of the function f , answer the parts (a)-(f) below.



(a) Find the domain of f . Express it in interval notation.

$$[-3, 3]$$

(b) Find the range of f . Express it in interval notation.

$$[0, 3]$$

(c) Find the x -intercepts.

$$(-1, 0), (1, 0)$$

(d) Find the y -intercepts.

$$(0, 1)$$

(e) Find the intervals on which f is increasing.

$$(-1, 0) \text{ and } (1, 3)$$

(f) Find the intervals on which f is decreasing.

$$(-3, -1) \text{ and } (0, 1)$$