MAC 1140, Fall 2017.

Exam #2

October 16, 2017

Name Key

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who uses a cell phone during the examination or if one is found within hands reach.
- Calculators are not allowed on this exam.
- The exam consist of two parts. Part I contains four multiple choice questions worth 6 points each. Part II contains four open ended questions worth 21.5 points each if not stated otherwise.

Part I

Choose your answer from five available choices. No partial credit will be given for wrong answers.

1. List potential rational zeros of the polynomial function $f(x) = 3x^4 - x^2 + 4x - 4$

   (a) $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}$

   (b) $1, 3, \frac{1}{2}, \frac{1}{4}, \frac{3}{2}, \frac{3}{4}$

   (c) $\pm 1, \pm 2, \pm 4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

   (d) $1, 2, 4, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}$

   (e) None of the above

   $-4: \pm 1, \pm 2, \pm 4$
   $3: \pm 1, \pm 3$
   $\dfrac{\pm 1, \pm 2, \pm 4}{1, 3}$

   $\pm 1, \pm 2, \pm 4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

2. Which is the following functions are polynomial functions

   - $f(x) = \frac{2}{3}x^4 - 1$ ✓
   - $g(x) = \frac{2-x}{x-1}$ ✗
   - $h(x) = \frac{2x^5}{5} - 3x^2 + 2x - 6$ $= \frac{2}{5}x^5 - 3x^2 + 2x - 6$ ✓
   - $k(x) = 3x - 2x^{1/2}$ ✗

   (a) f and h
   (b) f, h, and k
   (c) f, g, and k
   (d) f
   (e) None of the above

3. Find vertical asymptotes of the rational function

   $$f(x) = \frac{x^2 + x - 6}{(x-1)(x+3)} = \frac{\cancel{(x+3)}(x-2)}{(x-1)\cancel{(x+3)}} = \frac{x-2}{x-1}$$

   $x - 1 = 0$
   $\boxed{x = 1}$

   (a) $y = 1$
   (b) $x = 1$
   (c) $y = 1$ and $y = -3$
   (d) $x = 1$ and $x = -3$
   (e) None of the above

4. -3 and $-1 - 2i$ are zeros of a polynomial function. Which of the following is also a zero:

   (a) 3
   (b) $1 + 2i$
   (c) $-1 - 2i$
   (d) $-1 + 2i$
   (e) None of the above.

Part II

5. Graph the function $f(x) = \frac{x^3 - 4x}{x^2 - 1}$ $= \frac{x(x^2-4)}{(x-1)(x+1)} = \frac{x(x-2)(x+2)}{(x-1)(x+1)}$

(a) Domain

$\boxed{(-\infty,-1)\cup(-1,1)\cup(1,\infty)}$ or $\boxed{\{x \mid x \neq \pm 1\}}$

(b) y–intercept $f(0) = \frac{0}{-1} = 0$ $\boxed{(0,0)}$

(c) x–intercept

$x(x-2)(x+2) = 0$

$x = 0, \pm 2$ $\boxed{(0,0), (2,0), (-2,0)}$

(d) Vertical asymptote

$x = \pm 1$ $\boxed{\begin{matrix} x=1 \\ x=-1 \end{matrix}}$

(e) ~~Horizontal~~/oblique(slant) asymptote

$$\begin{array}{r|l} & \ \ x \\ \hline x^2-1 & x^3-4x \\ & x^3 \ \ -x \\ \hline & \ \ -5x \end{array}$$

$\boxed{y = x}$

(f) Intersection with asymptote

$\frac{x^3-4x}{x^2-1} = x$

$x^3-4x = x(x^2-1)$

$x^3-4x = x^3-x$

$-3x = 0$

$\boxed{x=0}$

$\boxed{(0,0)}$

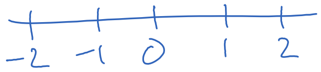
(g) Symmetries

$$f(-x) = \frac{(-x)^3 - 4(-x)}{(-x)^2 - 1} = \frac{-x^3 + 4x}{x^2 - 1} = \frac{-(x^3 - 4x)}{x^2 - 1}$$

$$= -\frac{x^3 - 4x}{x^2 - 1} = \underline{-f(x)} \quad \boxed{\text{odd}}$$

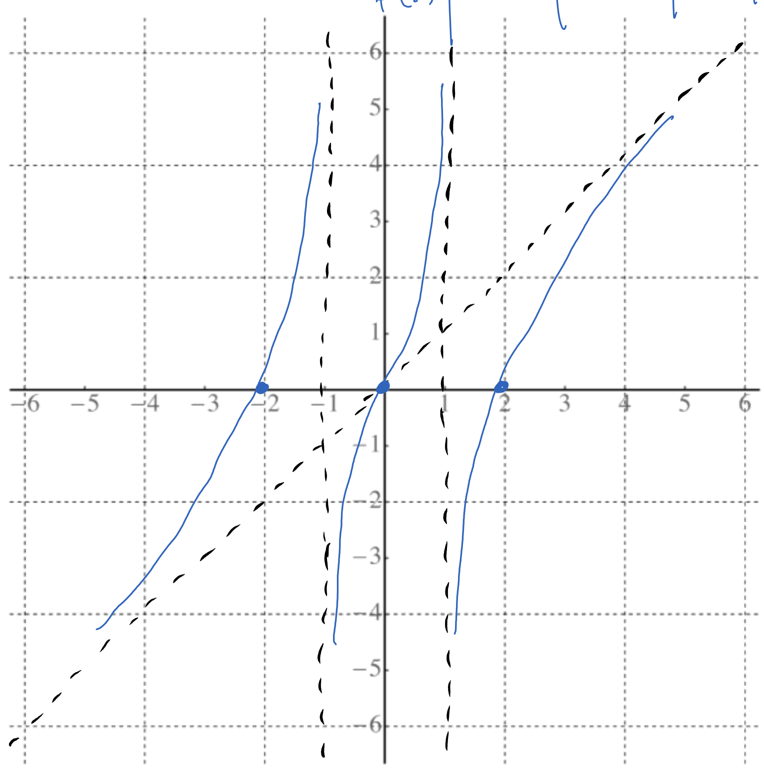
sym. with respect to origin

(h) Sign chart



	$(-\infty,-2)$	$(-2,-1)$	$(-1,0)$	$(0,1)$	$(1,2)$	$(2,\infty)$
pt	-3	-1.5	$-.5$	0.5	1.5	3
x	$-$	$-$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	$-$	$+$	$+$
$x+1$	$-$	$-$	$+$	$+$	$+$	$+$
$f(x)$	$-$	$+$	$-$	$+$	$-$	$+$

(i) Graph



6. Solve

$$x^3 = 9x - 10$$

$$x^3 - 9x + 10 = 0$$

let's find a zero using Rational Zero Thm:

$10: \pm1, \pm2, \pm5, \pm10$
$1: \pm1$
$\Bigg\} \pm1, \pm2, \pm5, \pm10$

$$\begin{array}{r|rrr|r} & 1 & 0 & -9 & 10 \\ 1 & & 1 & 1 & -8 \\ \hline & 1 & 1 & -8 & 2 \end{array} \quad \times$$

$$\begin{array}{r|rrr|r} & 1 & 0 & -9 & 10 \\ -1 & & -1 & 1 & 8 \\ \hline & 1 & -1 & -8 & 18 \end{array} \quad \times$$

$$\begin{array}{r|rrr|r} & 1 & 0 & -9 & 10 \\ 2 & & 2 & 4 & -10 \\ \hline & 1 & 2 & -5 & 0 \end{array} \quad \checkmark$$

$\boxed{x = 2}$

$$x^2 + 2x - 5 = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4\cdot(-5)}}{2} = \frac{-2 \pm \sqrt{4+20}}{2}$$

$$= \frac{-2 \pm \sqrt{4}\cdot\sqrt{6}}{2} = \frac{-2 \pm 2\sqrt{6}}{2} = \boxed{-1 \pm \sqrt{6}}$$

$$x = \boxed{\{2, -1 \pm \sqrt{6}\}}$$

7. Find the domain of $f(x) = \sqrt{2 - \frac{4}{x-3}}$

Solve: $2 - \frac{4}{x-3} \geq 0$

$$\frac{2x-6}{x-3} - \frac{4}{x-3} \geq 0$$

$$\frac{2x-10}{x-3} \geq 0$$

$$\frac{2(x-5)}{x-3} \geq 0$$

$2(x-5) = 0$ $\quad$ $x - 3 = 0$

$\underline{x = 5}$ $\quad$ $\underline{x = 3}$

3 $\quad$ 5

	$(-\infty, 3)$	$(3, 5]$	$[5, \infty)$
pt	0	4	6
$2(x-5)$	$-$	$-$	$+$
$x-3$	$-$	$+$	$+$
$f(x)$	$\boxed{+}$	$-$	$\boxed{+}$

Solution: $\boxed{(-\infty, 3) \cup [5, \infty)}$

8. Solve

$$2x^2 + 3 \geq 5x$$

$$2x^2 - 5x + 3 \geq 0$$

$$x = \frac{5 \pm \sqrt{25 - 4 \cdot 3 \cdot 2}}{2 \cdot 2} = \frac{5 \pm \sqrt{25 - 24}}{4}$$

$$= \frac{5 \pm \sqrt{1}}{4} = \begin{cases} \frac{5+1}{4} = \frac{6}{4} = \frac{3}{2} = \boxed{1.5} \\ \frac{5-1}{4} = \frac{4}{4} = \boxed{1} \end{cases}$$

$$2x^2 - 5x + 3 = 2(x - 1.5)(x - 1)$$

	$(-\infty, 1]$	$[1, 1.5]$	$[1.5, \infty)$
pt	0	1.2	2
$(x-1.5)$	−	−	+
$(x-1)$	−	+	+
	$\boxed{+}$	−	$\boxed{+}$

Solution: $\boxed{(-\infty, 1] \cup [1.5, \infty)}$