MAC 1140, Fall 2017.

Exam #5

December 4, 2017

Name ______________________________

- You will be told when to begin the work and when to terminate work on the examination. You must stop when instructed. Points may be deducted in case of violations.
- Please show your work to support your answers that require calculations. Correct but unsupported answers may not be given full credit.
- The use of a cell phone or other electronic communication devices during the examination is not allowed. The exam will be canceled and a grade of "0" will be assigned to anyone who uses a cell phone during the examination or if one is found within hands reach.
- Calculators are not allowed on this exam.
- The exam consist of two parts. Part I contains five multiple choice questions worth 5 points each. Part II contains 6 open ended questions.

Part I

Choose your answer from five available choices. No partial credit will be given for wrong answers.

1. Find the term containing x^9 in the expansion of $(x+3)^{12}$

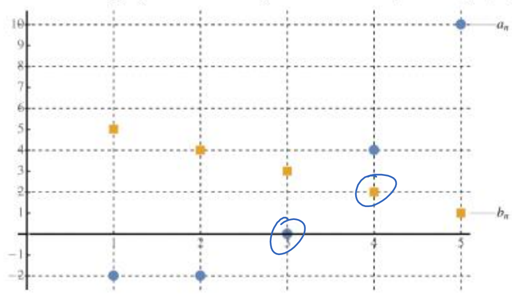
   (a) $\binom{12}{3}x^9 3^9$
   (b) $\binom{12}{9}x^9 3^3$
   (c) $\binom{12}{3}x^9 3^3$
   (d) $\binom{9}{12}x^9 3^3$
   (e) None of the above

2. Compute the binomial coefficient $\binom{6}{2}$

   $\frac{6!}{4!2!} = \frac{6\cdot 5\cdot \cancel{4!}}{\cancel{4!}\cdot 2\cdot 1} = 3\cdot 5 = 15$

   (a) 15
   (b) 20
   (c) 30
   (d) 40
   (e) None of the above

3. Given the graph of two sequences below, find $a_3 + b_4$.



   $0+2=2$

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) None of the above

4. Evaluate $\sum_{k=1}^{5} k(k-1)$ $= 1\cdot 0 + 2\cdot 1 + 3\cdot 2 + 4\cdot 3 + 5\cdot 4 = 2+6+12+20$
   $= 40$

   (a) 30
   (b) 35
   (c) 40
   (d) 50
   (e) None of the above

5. The sequence $\frac{1}{2}, 3, 18, 108, \ldots$ is

   $\frac{3}{\frac{1}{2}} = \frac{3}{1}\cdot\frac{2}{1} = \underline{\underline{6}}$ $\quad \frac{108}{18} = \underline{\underline{6}}$

   $\frac{18}{3} = \underline{\underline{6}}$

   (a) arithmetic
   (b) geometric
   (c) neither

Part II

6. (a) (10 pts) Find the general formula for $1, \underset{+3}{4}, \overset{+3}{7}, \underset{+3}{10}, 13, \ldots$

$$\boxed{a_n = 1 + 3(n-1)}$$

(b) (6 pts) Find a_{90}.

$$a_{90} = 1 + 3(90-1) = 1 + 3 \cdot 89 = \boxed{268}$$

(c) (10 pts) Find the sum $1 + 4 + 7 + 10 + 13 + \cdots + 121$.

$$121 = 1 + 3(n-1)$$
$$\frac{120}{3} = \frac{3(n-1)}{3}$$
$$40 = n-1$$
$$n = 41$$

$$S_{41} = \frac{41}{2} \cdot (1 + 121)$$
$$= \frac{41}{\cancel{2}} \cdot \frac{\cancel{122}^{61}}{1} = \boxed{41 \cdot 61}$$
$$= \boxed{2501}$$

7. (a) (10 pts) Given $a_3 = 4$ and $r = \frac{1}{2}$, find the 6th term of this sequence.

$$a_4 = 4 \cdot \tfrac{1}{2} = 2$$
$$a_5 = 2 \cdot \tfrac{1}{2} = 1$$
$$a_6 = 1 \cdot \tfrac{1}{2} = \boxed{\tfrac{1}{2}}$$

or use the gen. formula

(b) (8 pts) Write the sum of the first 20 terms or this sequence. Do not evaluate or simplify.

$$a_3 = 4 = a_1 r^{3-1}$$
$$4 = a_1 \cdot \left(\tfrac{1}{2}\right)^2$$
$$4 = a_1 \cdot \tfrac{1}{4}$$
$$16 = a_1$$

$$S_{20} = \boxed{\frac{16\left(1 - \left(\frac{1}{2}\right)^{20}\right)}{1 - \frac{1}{2}}}$$

8. (12 pts) Write the sum in sigma notation $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}+\cdots+\frac{10}{1024}$.

$$=\boxed{\sum_{i=1}^{10}\frac{i}{2^i}}$$

9. (12 pts) Expand $(2x-1)^5$ using the Binomial theorem. Any other method will result in zero pts.

$$=\binom{5}{0}(2x)^5(-1)^0+\binom{5}{1}(2x)^4(-1)^1+\binom{5}{2}(2x)^3(-1)^2+\binom{5}{3}(2x)^2(-1)^3+\binom{5}{4}(2x)^1(-1)^4+\binom{5}{5}(2x)^0\cdot(-1)^5$$

$$=1\cdot 32x^5\cdot 1+5\cdot 16x^4(-1)+10\cdot 8x^3\cdot 1+10\cdot 4x^2\cdot(-1)+5\cdot 2x\cdot 1+1\cdot 1\cdot(-1)$$

$$=\boxed{32x^5-80x^4+80x^3-40x^2+10x-1}$$

10. (7 pts) Find the 5th term of the sequence given by the recursive formula: $a_1=1, a_n=n\cdot a_{n-1}$.

$a_2=2\cdot 1=2$

$a_3=2\cdot 3=6$

$a_4=6\cdot 4=24$

$a_5=24\cdot 5=\boxed{120}$

11. Evaluate the expression

(a) (5 pts) $\frac{6!}{3!}$ $=\frac{6\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3!}}=\boxed{120}$

(b) (5 pts) $\frac{(n+2)!}{n!}$ $=\frac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}}=\boxed{(n+2)(n+1)}=\boxed{n^2+3n+2}$