Section 3.4

Ex: Solve:

$$x^4-6x^2-8x+24=0$$

24: ±1, ±2, ±4, ±6, ±12, ±24, ±3, ±8
1: ±1

Possible solutions: $\dfrac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24}{\pm1}$

	1	0	−6	−8	24
−1	↓	−1	1	5	3
	1	−1	−5	−3	27

Not sol.

	1	0	−6	−8	24
1	↓	1	1	−5	−13
	1	1	−5	−13	11

Not sol.

	1	0	−6	−8	24
2	↓	2	4	−4	−24
	1	2	−2	−12	0

$$x^4-6x^2-8x+24=0$$
$$(x-2)(x^3+2x^2-2x-12)=0$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{factor/solve}}$$

$-12: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$1: \pm 1$

	1	2	-2	-12
2		2	8	12
	1	4	6	0

$$(x-2)(x-2)\underbrace{(x^2+4x+6)}_{\text{quad. formula}}=0$$

$$x=\frac{-4\pm\sqrt{4^2-4\cdot 1\cdot 6}}{2\cdot 1}=\frac{-4\pm\sqrt{-8}}{2}$$

$$=\frac{-4\pm i\sqrt{8}}{2}=\frac{-\overset{2}{\cancel{4}}}{\cancel{2}}\pm i\,\frac{\cancel{\sqrt{4}}\cdot\sqrt{2}}{\cancel{2}}$$

$$=\boxed{-2\pm i\sqrt{2}} \leftarrow \text{complex solutions.}$$

solutions: $\{2, -2+i\sqrt{2}, -2-i\sqrt{2}\}$

Notes: 1) If a polynomial is of degree n, then counting multiple roots separately, the equation han n roots.

the equation has n roots.

2) If $a+bi$ is a root of a polynomial equation with real coefficients, then the complex number $a-bi$ $(b \neq 0)$ is a root also.

Solve: $x^4 - 6x^3 + 22x^2 - 30x + 13 = 0$

13: $\pm 1, \pm 13$
1: ± 1
$\pm 1, \pm 13$

	1	-6	22	-30	13
1	↓	1	-5	17	-13
	1	-5	17	-13	0

$\boxed{x=1}$

$x^3 - 5x^2 + 17x - 13 = 0$

	1	-5	17	-13
1		1	-4	13
	1	-4	13	0

$\boxed{x=1}$

$x^2 - 4x + 13 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 13}}{2} = \frac{4 \pm \sqrt{16-52}}{2}$$

$$= \frac{4}{2} \pm \frac{\sqrt{-36}}{2} = 2 \pm \frac{6i}{2} = \boxed{2 \pm 3i}$$

solutions: $\{1, 2-3i, 2+3i\}$

<u>Thm:</u> If $f(x)$ is a polynomial of degree n, $n \geq 1$, then the equation $f(x)=0$ has at least one complex root.

(Fundamental thm of Algebra)

<u>Ex:</u> Find a polynomial with real coefficients, so that $-2, 2, i$ are zeros and $f(3)=-150$.

Also $-i$ is a root.

$$(x-2)(x+2)(x-i)(x+i) = (x^2-4)(x^2-i^2)$$
$$= (x^2-4)(x^2+1) = x^4+x^2-4x^2-4$$
$$= x^4-3x^2-4$$

$$f(x) = A \cdot (x^4-3x^2-4)$$

$f(3) = -150$

$$f(x) = A\cdot(x^4 - 3x^2 - 4)$$

$$f(3) = -150$$

$$A(3^4 - 3\cdot 3^2 - 4) = -150$$

$$A(81 - 27 - 4) = -150$$

$$A\cdot 50 = -150$$

$$\boxed{A = -3}$$

$$f(x) = -3(x^4 - 3x^2 - 4) = \boxed{-3x^4 + 9x^2 + 12}$$

Thm: (Descarte's Rule of Sign)

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ be a polynomial real coefficients.

1) The number of positive real zeros of f is either:
   - the same as the number of sign changes of $f(x)$

     or

   - less than the number of sign changes of $f(x)$ by a positive even number.

2) The number of negative real zeros of f is either:
   - the same as the number of sign changes

is either:

- the same as the number of sign changes of $f(-x)$ or
- less that the number of sign changes of $f(-x)$ by a positive even number.

<u>Ex:</u> $f(x) = 3x^7 - 2x^5 - x^4 + 7x^2 + x - 3$

sign change

3 sign changes of $f(x) \to$ the number of positive roots is 3 or 1.

$f(-x) = -3x^7 + 2x^5 - x^4 + 7x^2 - x - 3$

4 sign changes of $f(-x) \to$ the number of negative roots is 4, 2, 0.