10/04

Wednesday, October 4, 2017 11:00 AM

Offline HW 4 due 10/9

Exam 2 on 10/16

Section 3.4

Ex: Solve: $x^3+2x^2+5x+4=0$

$f(x) = x^3+2x^2+5x+4 \rightarrow$ 0 changes in signs

No positive roots

$f(-x) = (-x)^3+2(-x)^2+5(-x)+4$

$= -x + 2x - 5x + 4 \rightarrow$ 3 changes in signs

3, 1 negative roots

4: $\pm 1, \pm 2, \pm 4$

1: ± 1

possible rational roots: $\pm 1, \pm 2, \pm 4$

Using the Descarthe's thm. we can eliminate 1, 2, 4.

Test: $\boxed{-1}$

	1	2	5	4
-1	$\downarrow$	-1	-1	-4
	1	1	4	0

$$\boxed{x^2+x+4=0}$$

Test -1:

	1	1	4
-1	$\downarrow$	-1	0
	1	0	4

$\neq 0$ not a root

Test -2:

	1	1	4
-2		-2	2
	1	-1	6

$\neq 0$ not a root

Test -4:

	1	1	4
-4	$\downarrow$	-4	12
	1	-3	16

not a root

$$x^2+x+4=0$$

Use quadratic formula:

$$x=\frac{-1\pm\sqrt{(1)^2-4\cdot 1\cdot 4}}{2\cdot 1}=\frac{-1\pm\sqrt{1-16}}{2}$$

$$=\frac{-1\pm\sqrt{-15}}{2}=\frac{-1\pm i\sqrt{15}}{2}$$

The solutions are: $\boxed{-1,\ -\frac{1}{2}+\frac{i\sqrt{15}}{2},\ -\frac{1}{2}-\frac{i\sqrt{15}}{2}}$

real complex

Section 3.5

Def: A rational function can be expressed as

$$\frac{p(x)}{q(x)},$$

where p and q are polynomials.

Ex:

function	$\frac{x^2-x}{x+3}$	$\frac{x}{\sqrt{2}\cdot x}$	$\frac{x^5-1}{\sqrt{x}+3}$	x^5+3x-2
rational?	yes	yes	NO	yes

Note: To find the domain of $\frac{p(x)}{q(x)}$, we have to find all real solutions of $q(x)=0$.

Ex: Find the domain:

- $f(x)=\dfrac{x^2-9}{x+3}$

$$x+3=0$$
$$x=-3$$

Domain: $(-\infty,-3)\cup(-3,\infty)$

- $g(x)=\dfrac{x}{x^2-9}$

$$x^2-9=0$$
$$(x-3)(x+3)=0$$
$$x=\pm 3$$

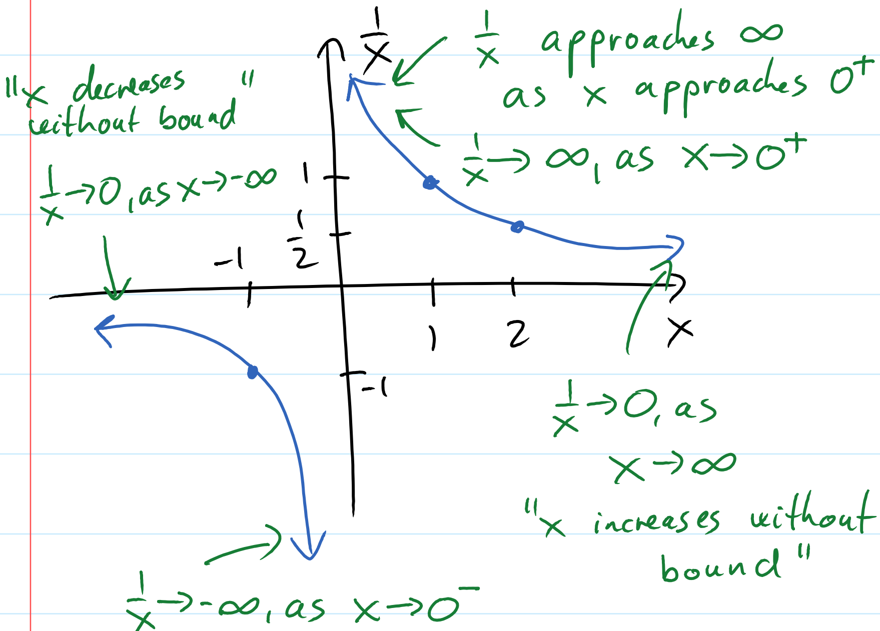
Dom: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

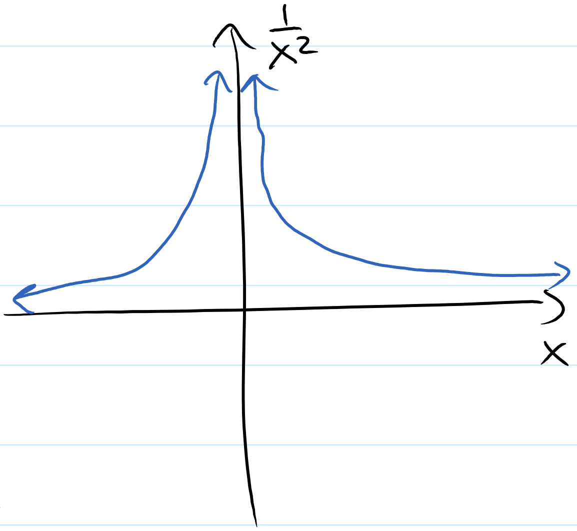
- $h(x)=\dfrac{x-3}{x^2-9}$

$$x^2-9=0$$
$$(x-3)(x+3)=0$$
$$x=\pm 3$$

Dom: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

Basic rational functions are $\frac{1}{x}$, $\frac{1}{x^2}$





Vertical asymptote

Vertical asymptote

$f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ have no common factors.

The line $x=a$ is a vertical asymptote of $f(x)$ if $q(a)=0$. ($p(a) \neq 0$)

Ex: Find all vertical asymptotes:

- $f(x) = \frac{x}{x^2-9} = \frac{x}{(x-3)(x+3)}$ $\quad (x-3)(x+3)=0$

  $x = \pm 3$ ver. asym.

- $g(x) = \frac{x+3}{x^2-9} = \frac{\cancel{x+3}}{(x-3)\cancel{(x+3)}} = \frac{1}{x-3}$,

  $x-3=0$

  $x=3$ ver. asym.

Horizontal asymptote

The line $y=b$ is a horizontal asymptote of the graph of $f(x)$ if $f(x)$ approaches b

of the graph of $f(x)$ if $f(x)$ approaches b as x increases or decreases without bound.

$$(f(x) \to b, \text{ as } x \to \infty \text{ or } x \to -\infty)$$

Does $\frac{1}{x}$ have a hor. asymptote?

yes: $y = 0$.