Survey reflection:

- Math only tutoring in ACl-379A
  - M: 11AM-3PM
  - Tu: 8AM-7PM
  - W: none
  - Th: 8AM-7PM
  - F: 1PM-7PM

- The following are FIU wide policies:
  - HW and Quiz load and due dates
  - 80% HW completion requirement for Quizzes.
  - No calculator policy on exams

- MyLabsPlus contains
  - your (updated) grades
  - syllabus
    - my office hours
    - exam & offline HW due dates
  - class notes

Offline HW 4 due Monday

HW review:

Find 3rd degree polynomial, with the following zeros: -5, $4+3i$, and $f(2)=91$.

Since $4+3i$ is a zero, then $4-3i$ is a zero

$$f(x) = A\cdot(x-(-5))(x-(4+3i))(x-(4-3i))$$
$$= A\cdot(x+5)(\underbrace{x-4}_{a}\ \underbrace{-3i}_{b})(\underbrace{x-4}_{a}\ \underbrace{+3i}_{b})$$
$$(a-b)\cdot(a+b) = a^2-b^2$$

$$= A(x+5)((x-4)^2-(3i)^2)$$
$$= A(x+5)(x^2-8x+16-\underbrace{9(-1)}_{3^2\cdot i^2})$$

$$= A(x+5)(x^2-8x+25)$$
$$= A(x^3-8x^2+25x+5x^2-40x+125)$$
$$= A(x^3-3x^2-15x+125)$$

$f(2)=91$: $f(2) = \underbrace{A\cdot(8-3\cdot4-30+125) = 91}_{\text{Solve for } A.}$

Section 3.5 cont. (Rational functions and their graphs)

Finding horizontal asym

Let f be the rational function given by

deg

Let f be the rational function given by

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0}, \quad \begin{array}{|c} \text{deg} \\ \hline n \\ \hline m \end{array}$$

$(a_n \neq 0,\ b_m \neq 0)$

1) If $n < m$, the line $y = 0$ is a hor. asym.
2) If $n > m$, there is no horizontal asymptote.
3) If $n = m$, the line $y = \frac{a_n}{b_m}$ is a hor. asymptote.

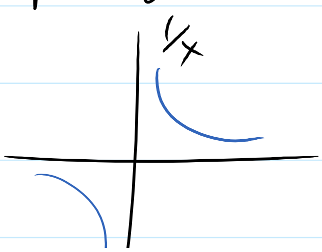
Ex: Find the hor. asym:

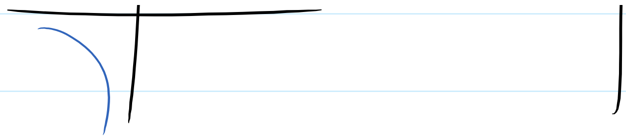
- $f(x) = \dfrac{4x}{2x^2+1}$ $\quad \left.\begin{array}{|c} \text{deg} \\ \hline 1 \\ \hline 2 \end{array}\right\} \boxed{y=0}$

- $f(x) = \dfrac{4x^2}{2x^2+1}$ $\quad \left.\begin{array}{|c} \text{deg} \\ \hline 2 \\ \hline 2 \end{array}\right\} y = \frac{4}{2} \Rightarrow \boxed{y=2}$

- $g(x) = \dfrac{4x^2(1-x)}{2x^2+1}$ $\quad \left.\begin{array}{|c} \text{deg} \\ \hline 3 \\ \hline 2 \end{array}\right\} \boxed{\text{no hor. asymp}}$

Basic rational functions:





Sketch: $y = \frac{1}{(x-2)^2} + 1$

Use transformation

$\frac{1}{x^2} \longrightarrow$ hor. shift to right by 2 units $\longrightarrow \frac{1}{(x-2)^2} \longrightarrow$ vert. shift up by 1 unit. $\longrightarrow \frac{1}{(x-2)^2} + 1$

Ex: Graph: $f(x) = \frac{2x-1}{x-1}$

1) symmetry: $f(-x) = f(x)$ y-axis sym.

$f(-x) = -f(x)$ origin sym.

$$f(-x) = \frac{2(-x)-1}{(-x)-1} = \frac{-2x-1}{-x-1}\frac{(-1)}{(-1)} = \frac{2x+1}{x+1}$$ No symmetry

$$-f(x) = -\frac{2x-1}{x-1} = \frac{-(2x-1)}{x-1} = \frac{-2x+1}{x-1}$$

$$= \frac{2x-1}{-(x-1)} = \frac{2x-1}{-x+1}$$

2) Find the y-intercept: $f(0)$

$$f(0) = \frac{2\cdot 0-1}{0-1} = \frac{-1}{-1} = 1 \qquad \boxed{(0,1)}$$

3) Find the x-intercept(s): $f(x)=0$

$$\frac{2x-1}{x-1}=0 \rightarrow 2x-1=0$$

$$2x=1$$

$$x=\frac{1}{2} \qquad \boxed{\left(\frac{1}{2},0\right)}$$

4) Find vertical asym: "simplify and set the denom.=0"

$$\frac{2x-1}{x-1} \rightarrow x-1=0$$

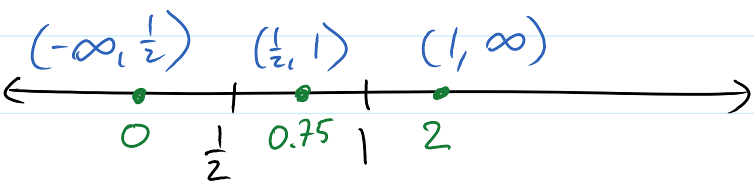
$$\boxed{x=1}$$

5) Find hor. asym:

$$\left.\begin{array}{l}\text{deg of num: } 1\\ \text{deg of den: } 1\end{array}\right\} y=\frac{2}{1}$$

$$\boxed{y=2}$$

6) Plot at least one pt. between each x-int. and vert. asym. (Find the sign graph)



let's find the sign (pos. or neg) of $f(x)$ on: $\underbrace{\left(-\infty,\frac{1}{2}\right)}_{0}$, $\underbrace{\left(\frac{1}{2},1\right)}_{0.75}$, $\underbrace{(1,\infty)}_{2}$

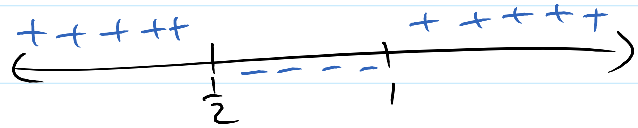
$f(x)=\frac{2x-1}{x-1}$

$$f(0)=\frac{-1}{-1}=1>0$$

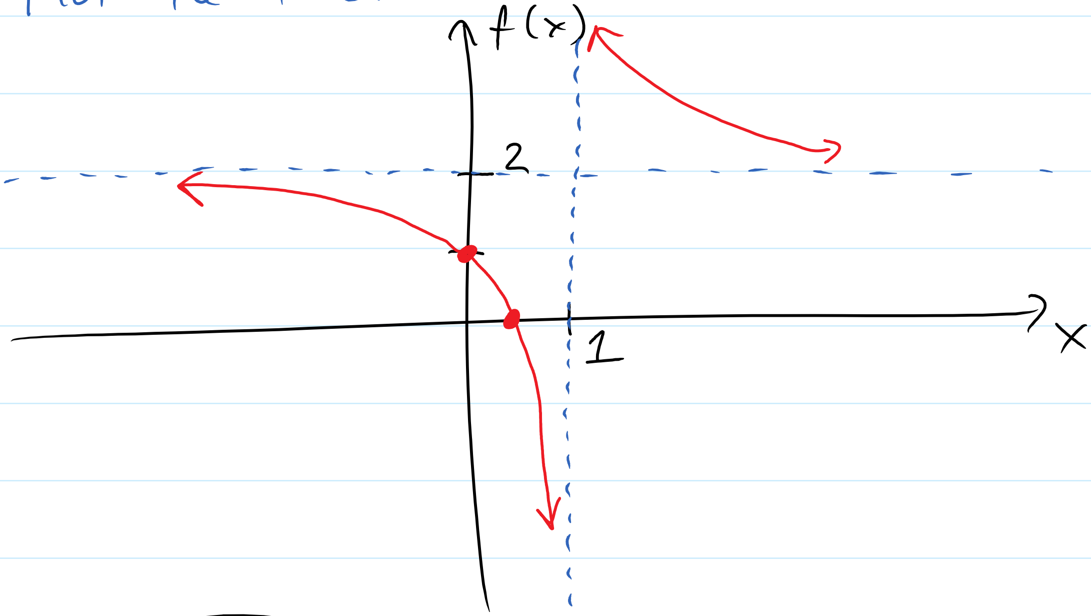
$$f(0.75)=\frac{2\cdot 0.75-1}{\ldots}=\frac{1.5-1}{\ldots}<0$$

$$f(0.75) = \frac{2 \cdot 0.75 - 1}{0.75 - 1} = \frac{1.5 - 1}{-0.25} < 0$$

$$f(2) = \frac{2 \cdot 2 - 1}{2 - 1} > 0$$



7) Plot the func:



Slant asym.

If the degree of numerator is one more than the deg. of denom. the graph has slant asym.

$$\frac{x^2+1}{x-1}$$

$$\begin{array}{r|l} & \quad x + 1 \\ \hline x-1 & x^2 + 0x + 1 \\ & -x^2 + x \\ \hline & \qquad x + 1 \\ & \quad -x + 1 \\ \hline & \qquad\quad 2 \end{array}$$

slant asym: $y = x + 1$

$y = x+1$

2

$$\frac{x^2+1}{x-1} = x+1 + \frac{2}{x-1}$$