

Exam 2:

- 4 multiple choice questions
- 4 open ended questions
- sections: 3.3 - 3.6

Reviews:

- Saturday, 10/14, 10AM-noon, MMC PCA 135
- Sunday 8-9PM, online

Review:

Solve: $x^3 + 9x^2 + 16x = 6$

$$-6 \quad -6$$

$$x^3 + 9x^2 + 16x - 6 = 0$$

$$\left. \begin{array}{l} -6: \pm 1, \pm 2, \pm 3, \pm 6 \\ 1: \pm 1 \end{array} \right\} \pm 1, \pm 2, \pm 3, \pm 6$$

Descartes's rules:

$$f(x) = x^3 + 9x^2 + 16x - 6 \quad 1 \text{ change} \rightarrow 1 \text{ pos. root}$$

$$f(-x) = -x^3 + 9x^2 - 16x - 6 \quad 2 \text{ changes} \rightarrow 2 \text{ or } 0 \text{ neg. roots}$$

$x^2 + 6x - 2 = 0$ Δ changes \rightarrow Δ or \cup neg. roots

	1	9	16	-6
-3	↓	-3	-18	6
	1	6	-2	0

}

x = -3 is a solution

$$x^2 + 6x - 2 = 0$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2 \cdot 1} = \frac{-6 \pm \sqrt{36 + 8}}{2}$$

$$= \frac{-6 \pm \sqrt{44}}{2} = \frac{-6 \pm 2 \cdot \sqrt{11}}{2} = \frac{-6}{2} \pm \frac{2\sqrt{11}}{2}$$

$$= \boxed{\frac{-6 \pm \sqrt{44}}{2}} \begin{cases} \boxed{-3 + \sqrt{11}} \approx -3 + 3.3 = 0.3 \\ \boxed{-3 - \sqrt{11}} \approx -3 - 3.3 = -6.3 \end{cases}$$

$\underbrace{\quad}_{2}$
 $\underbrace{\quad}_{3.3}$

Section 4.1 (Exponential functions)

Def: The exponential function f with base b is defined as

$$f(x) = b^x \quad \text{or} \quad y = b^x,$$

where b is a positive constant other than 1
and x can be any real number

$$\text{domain} = (-\infty, \infty)$$

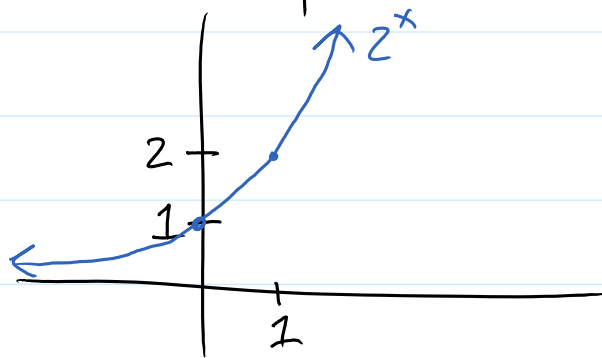
Ex: $y = 2^x$ or $f(x) = 2^x$

let's graph it

$$x^{-a} = \frac{1}{x^a}$$

$$x^0 = 1 \quad (\text{even if } x=0)$$

x	-10	-2	-1	0	1	2	4
$f(x)$	2^{-10}	2^{-2}	$\frac{1}{2}$	2^0	2	4	16
	"	"	"	"	"	"	"
	$\frac{1}{2^{10}}$	$\frac{1}{2^2}$	"	1	"	"	"
	"	$\frac{1}{4}$	"	"	"	"	"



$$\text{Dom: } (-\infty, \infty)$$

$$\text{Range: } (0, \infty)$$

$$\text{horizontal asym: } y=0$$

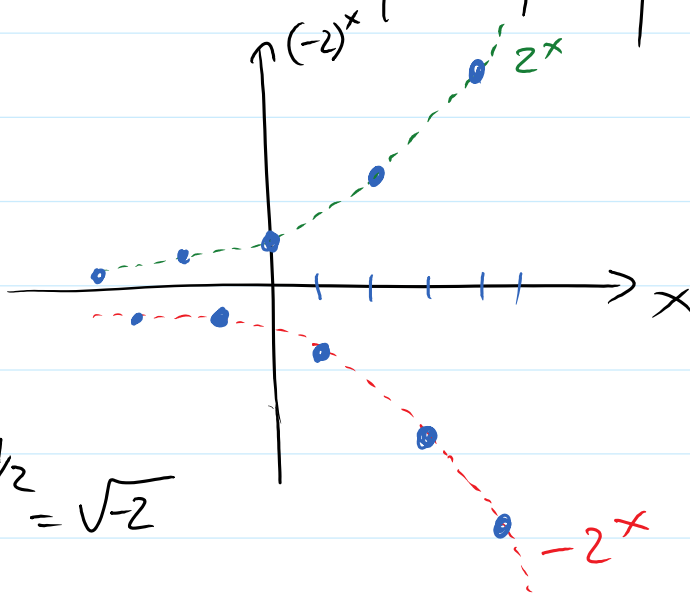
What if $b=1$ or is negative?

$f(x) = 1^x = 1$ ← constant function, not an exponential.

x	-2	-1	0	1	2
$f(x)$	1^{-2} " " $\frac{1}{2}$ " " 1	1	1^0 " " 1	1	1

$f(x) = (-2)^x$

x	0	1	2	3	4	5
$f(x)$	1	-2	4	-8	16	-32



$f(\frac{1}{2}) = (-2)^{\frac{1}{2}} = \sqrt{-2}$

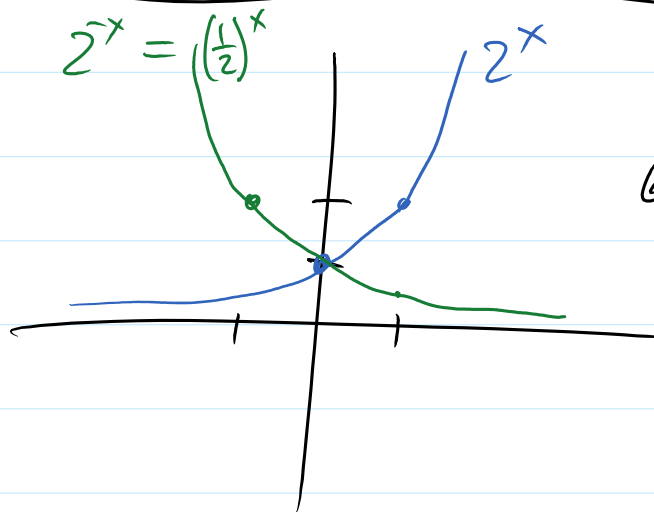
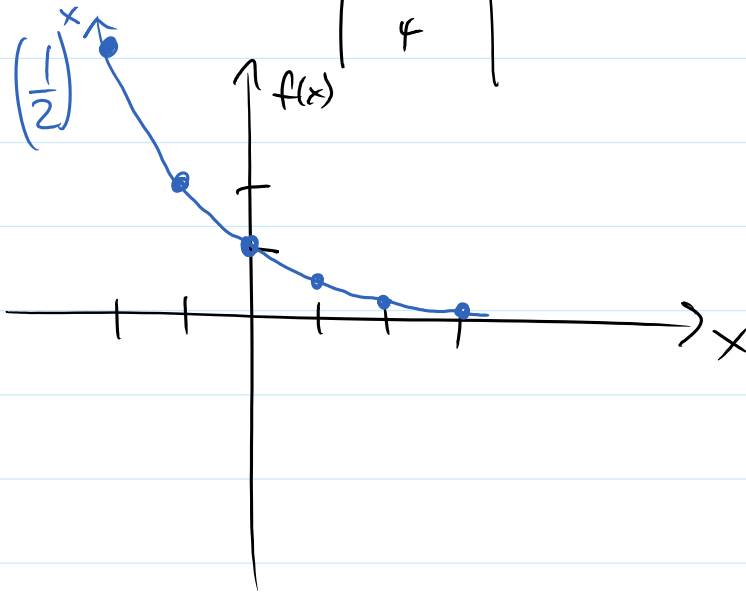
Ex: Graph $f(x) = (\frac{1}{2})^x$

Alg. rules

$$\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^c$$

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

x	-2	-1	0	1	2	3
f(x)	$\left(\frac{1}{2}\right)^{-2}$	$\left(\frac{1}{2}\right)^{-1}$	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
	$\left(\frac{2}{1}\right)^2$	$\left(\frac{2}{1}\right)^1$				
	2 ²	2				
	4					



what's the relationship?

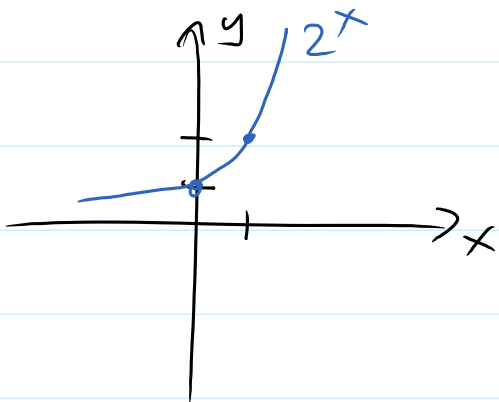
Simplify: $\left(\frac{1}{2}\right)^x = \frac{1^x}{2^x} = \frac{1}{2^x} = 2^{-x}$

$f(x) = 2^x$, what transformation gives:
 $f(-x) = 2^{-x}$

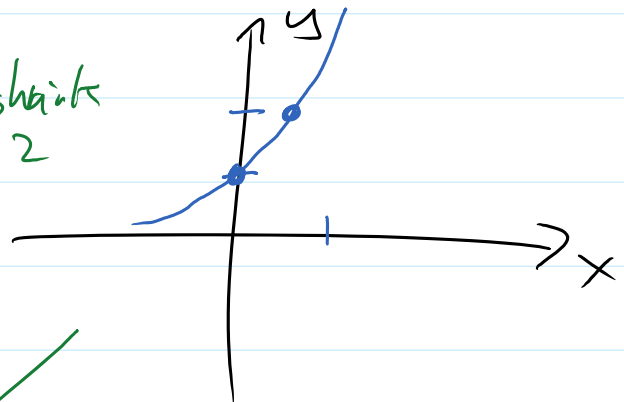
↖ reflection about the y-axis

Ex: Use the transft. to graph

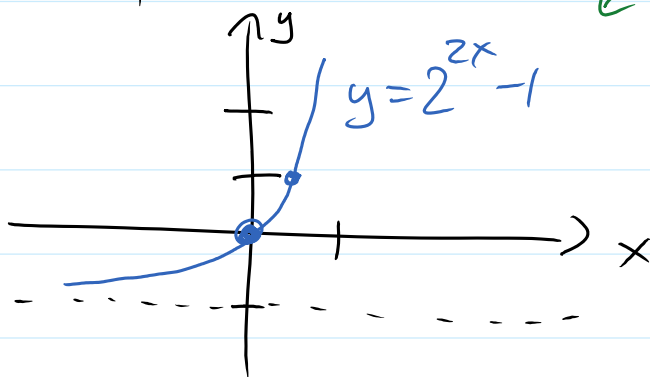
② ← hor. shrink
 $f(x) = 2^{2x} - 1$
down 1



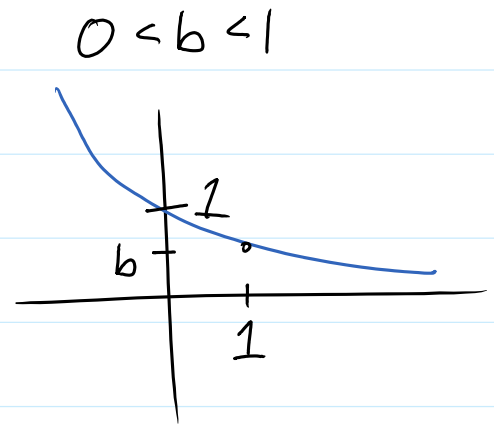
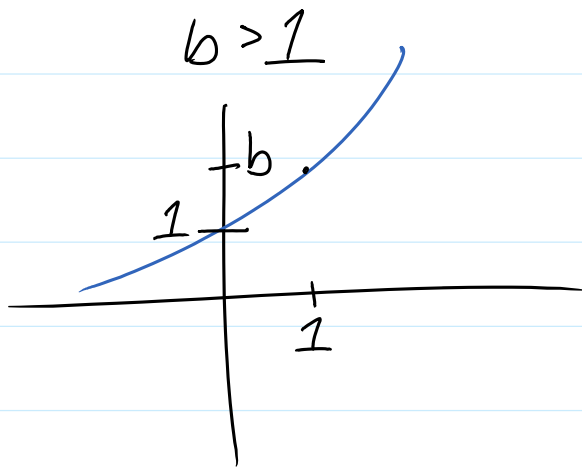
hor. shrink by 2 →



↙ down by 1

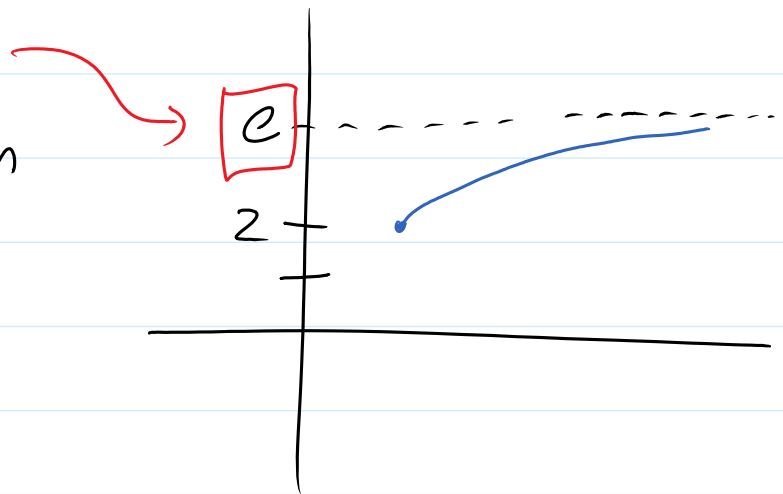


The graph of $y = b^x$ is



Euler constant

$$f(n) = \left(1 + \frac{1}{n}\right)^n$$



$$e \approx 2.718\dots$$

Plot $f(x) = e^x$ (natural exponential func.)
(e is natural base)

