10/14

Saturday, October 14, 2017 10:02 AM

Section 3.3

- $(4x^2-8x+6) \div (2x-1)$

long division

$$\frac{4x^2}{2x} = 2x \qquad \frac{-6x}{2x} = -3$$

$$
\begin{array}{r l}
 & 2x - 3 \\
2x-1 \,\big) & 4x^2 - 8x + 6 \\
 & -4x^2 + 2x \downarrow \\
\hline
 & -6x + 6 \\
 & +6x \mp 3 \\
\hline
 & 3
\end{array}
$$

$3 \leftarrow$ remainder

- $((12x^3+6x^2-6x+10) \div (3x^2+5)$

$$\frac{12x^3}{3x^2} = 4x \qquad \frac{6x^2}{3x^2} = 2$$

$$
\begin{array}{r l}
 & 4x + 2 \\
3x^2+5 \,\big) & 12x^3 + 6x^2 - 6x + 10 \\
 & -12x^3 \downarrow \mp 20x \\
\hline
 & 6x^2 - 26x + 10 \\
 & -6x^2 \quad \mp 10 \\
\hline
\end{array}
$$

$$\begin{array}{r} -6x^2 \qquad \mp 10 \\ \hline -26x \end{array} \leftarrow \text{remainder}$$

$$\frac{12x^3+6x^2-6x+10}{3x^2+5} = 4x+2 + \frac{-26x}{3x^2+5}$$

$$12x^3+6x^2-6x+10 = (4x+2)(3x^2+5) - 26x$$

- Solve: $2x^3-5x^2+x+2=0$ given that 2 is a zero of $f(x) = 2x^3-5x^2+x+2$

$$(2x^3-5x^2+x+2) \div (x-2)$$

$$\begin{array}{c|cccc} & 2 & -5 & 1 & 2 \\ 2 & \downarrow & 4 & -2 & -2 \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

Find the zeros of $2x^2-x-1$

Let's try: ± 1

-1

$$\begin{array}{c|ccc} & 2 & -1 & -1 \\ -1 & \downarrow & -2 & 3 \\ \hline & 2 & -3 & 2 \end{array}$$

1

$$\begin{array}{c|ccc} & 2 & -1 & -1 \\ 1 & \downarrow & 2 & 1 \\ \hline & 2 & 1 & 0 \end{array}$$

$|2\ |-3\ |\ 2$ $\qquad$ $|\ 2\ |\ 1\ |\ \underline{\underline{0}}$

$x = 1$ is a zero.

Find the zero of $2x + 1 = 0$

$$\begin{aligned} 2x + 1 &= 0 \\ -1 \quad & \ -1 \\ \frac{2x}{2} &= \frac{-1}{2} \\ x &= \boxed{-1/2} \end{aligned}$$

Zeros: $\{-\frac{1}{2}, 1, 2$.

Section 3.4

list all possible rational zeros:

- $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

divisors of 15: $\pm 1, \pm 3, \pm 5, \pm 15$

divisors of 2: $\pm 1, \pm 2$

possible zeros: $\dfrac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1, \pm 2}$

$$= \left\{ \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2} \right\}$$

- $f(x) = x^4 + 2x^5 - 7x^2 + 12x - 12$

$$= \underline{2}x^5 + x^4 - 7x^2 + 12x \underline{-12}$$

$-12: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$2: \pm 1, \pm 2$

$$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \overset{\pm 1 \,=}{\pm \frac{2}{2}}, \pm \frac{3}{2}, \overset{\pm 2 \,=}{\pm \frac{4}{2}}, \overset{\mp 3 \,=}{\pm \frac{6}{2}}, \underset{= \pm 6}{\pm \frac{12}{2}}$$

Solution $\left\{ \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2} \right\}$

Solve:

- $\underline{x}^3 - 2x^2 - 7x \underline{-4} = 0$

$-4: \pm 1, \pm 2, \pm 4$

$1: \pm 1$

possible zeros: $\pm 1, \pm 2, \pm 4$

Test 1:

	1	-2	-7	-4
1		1	-1	-8
	1	-1	-8	-12

Test -1:

	1	-2	-7	-4
-1	↓	-1	3	4
	1	-3	-4	0

1	-1	-8	-12

$\boxed{\text{NO}}$

1	-3	-4	0

yes: $x=-1$ is a zero.

Solve: $x^2-3x-4=0$

$(x-4)(x+1)=0$

$x-4=0 \qquad x+1=0$

$x=4 \qquad x=-1$

Solution: $\{-1, 4\}$

Solve: $x^4-2x^2-16x-15=0$

zeros: $\pm1, \pm3, \pm5, \pm15$

Test 1:

	1	0	-2	-16	-15
1	↓	1	1	-1	-17
	1	1	-1	-17	-32

Test -1:

	1	0	-2	-16	-15
-1	↓	-1	1	1	15
	1	-1	-1	-15	0

$\boxed{x=-1}$

now: $x^3-x^2-x-15=0$

Test -1:

possible zeros: $-1, \pm3, \pm5, \pm15$

	1	-1	-1	-15

Test +3:

	1	−1	−1	−15
−1	↓	−1	2	−1
	1	−2	1	−16

NO

Test +3:

	1	−1	−1	−15
3	↓	3	6	15
	1	2	5	0

$\boxed{x=3}$ is a solution

now solve:

$$x^2+2x+5=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{4-4\cdot1\cdot5}}{2}$$

$$=\frac{-2\pm\sqrt{4-20}}{2}=\frac{-2\pm\sqrt{-16}}{2}=\frac{-2\pm i\sqrt{16}}{2}$$

$$=\frac{-2\pm i\cdot 4}{2}=\frac{-2}{2}\pm\frac{4i}{2}=\boxed{-1\pm 2i}$$

Solutions: $\{-1\pm 2i, 3, -1\}$

Find 3rd degree pol. that has −5 and 4+3i as zeros and $f(2)=91$

complex conjugate

zeros: −5, 4+3i, 4−3i

$f(x) = A\,(x-(-5))\,(x-(4+3i))\,(x-(4-3i))$

$= A\,(x+5)\,(\underbrace{x-4}_{(a}\;\underbrace{-3i}_{-\,b)})\,(\underbrace{x-4}_{(a}\;\underbrace{+3i}_{+\,b)}) = a^2-b^2$

$= A\,(x+5)\,((x-4)^2-(3i)^2)$

$= A(x+5)\,(x^2-2\cdot x\cdot 4+16-9i^2)$

$= A(x+5)\,(x^2-8x+16+9)$

$= A\,(x+5)\,(x^2-8x+25)$

$= A(x^3-8x^2+25x+5x^2-40x+125)$

$= A(x^3-3x^2-15x+125)$

To find A, use $f(2)=91$

$$f(2)=91$$

$A(8-3\cdot 4-15\cdot 2+125)=91$

$A(8-12-30+125)=91$

$A(-4+95)=91$

$\dfrac{A(91)}{91} = \dfrac{91}{91} \longrightarrow A = \dfrac{91}{91} = \underline{1}$

$$\boxed{f(x)=x^3-3x^2-15x+125}$$

Section 3.5

1) Find Domain

2) vert. asym.

3) hor. asym. / slant (oblique) asym

solve:

3) hor. asym. / slant (oblique) asym

4) find the int. with asym   solve: f(x) = eq. of the asym.

5) y- and x-intercept

6) sign graph

7) graph the function

- $f(x) = -\dfrac{1}{x^2-4}$

1) $x^2-4 \neq 0$

$(x-2)(x+2) \neq 0$

$x \neq \pm 2$

$$\boxed{\begin{array}{l}(-\infty,-2)\cup(-2,2)\cup(2,\infty)\\ \{x \mid x \neq \pm 2\}\end{array}}$$

2) $\dfrac{-1}{x^2-4} = \dfrac{-1}{(x-2)(x+2)}$

$(x-2)(x+2) = 0$

$\boxed{x = \pm 2}$

3) deg numerator is 0, deg. of denom. is 2

$0 < 2 \rightarrow \boxed{y=0}$ is a hor. asym.

4) $\dfrac{-1}{x^2-4} = 0$

n - deg of denom

m - deg. of num.

x^2-4

$-1=0$ $\boxed{\text{none}}$

m - deg. of num.

	asym.
$n=m$	hor. asym. is $y=$ "ratio of leading coeff"
$n>m$	hor. asym. is $y=0$
$n<m$	no horizontal asym
$n+1=m$	slant (oblique) asym.

5) y-int

$f(0)=\frac{-1}{0-4}=\frac{1}{4}$

$\boxed{(0, \frac{1}{4})}$

x-int

$f(x)=0$ $\boxed{\text{no x-int}}$

$\frac{-1}{x^2-4}=0$

$-1=0$ no solution

6) $- - - - \quad -2 \quad + + + + + + + \quad 2 \quad - - - -$

$-3 \quad -2 \quad 0 \quad 2 \quad 3$

x-int, vert. asym.

$f(-3)=\frac{-1}{(-3)^2-4}=\frac{-1}{9-4}=\frac{-1}{5}<0$

$f(0)=\frac{-1}{0-4}=\frac{-1}{-4}=\frac{1}{4}>0$

$f(3)=\frac{-1}{3^2-4}=\frac{-1}{5}<0$

6.5) Symmetric

$$f(-x) = \frac{-1}{(-x)^2-4} = \frac{-1}{x^2-4} = f(x)$$

odd

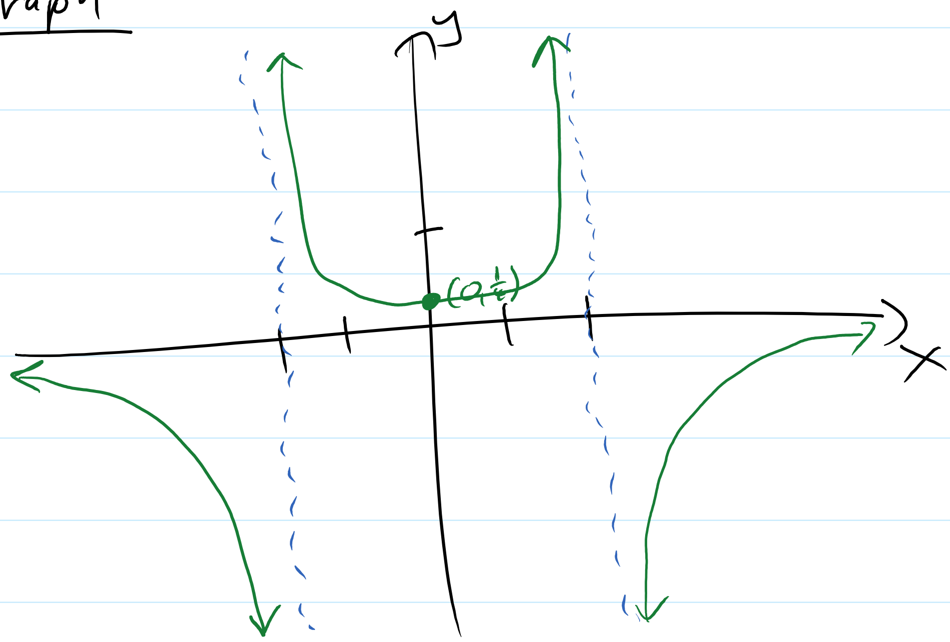
$f(-x) = -f(x)$

even

$f(-x) = f(x)$

$\boxed{\text{even}}$ symm. with respect to y-axis

7) Graph



Graph: $\dfrac{x^2-x-6}{x-3}$

1) Domain: $x \neq 3$

$\boxed{(-\infty,3)\cup(3,\infty)}$

2) $$\frac{x^2-x-6}{x-3} = \frac{(\cancel{x-3})(x+2)}{\cancel{x-3}} = \frac{x+2}{1}$$

vert. asy: $1=0$

no solution $\rightarrow$ $\boxed{\text{no vertical asym}}$

no solution → $\boxed{\text{no vertical asym}}$

there is a "hole" at $x=3$

3) deg of num > deg of denom. → $\boxed{\text{no hor. asym}}$

slant (oblique): $(x^2-x-6)\div(x-3) = \boxed{x+2} + \dfrac{0}{x-3}$

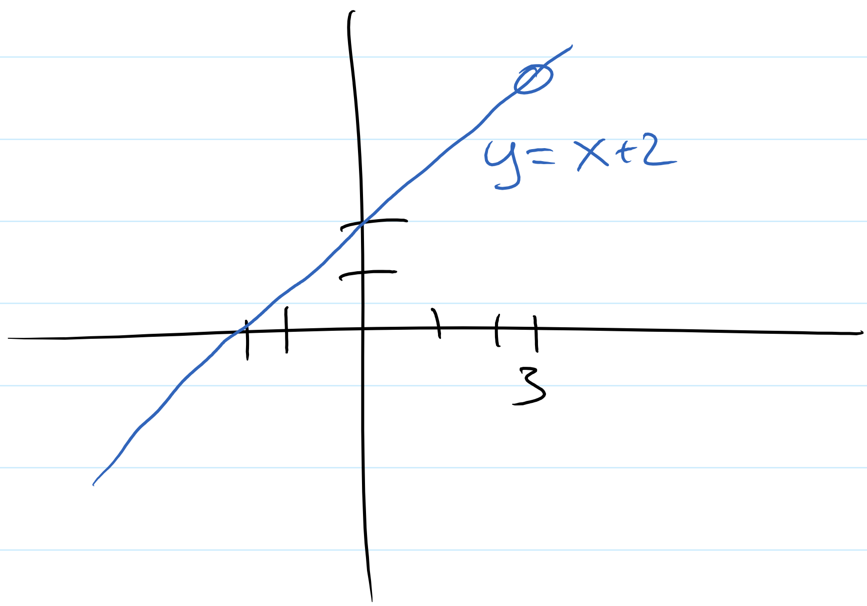
$\boxed{y=x+2}$

4) $\dfrac{x^2-x-6}{x-3} = x+2 \qquad // \cdot (x-3)$

$x^2-x-6 = (x+2)(x-3)$

$x^2-x-6 = x^2-x-6$

$0=0$ $\boxed{\text{True for all } x}$



Section 3.6

Find the domain of:

$$f(x) = \sqrt{\frac{x}{2x-1} - 1}$$

since the inside of sqrt has to be ≥ 0

$$\frac{x}{2x-1} - 1 \geq 0$$

$$\frac{x}{2x-1} - \frac{1\cdot(2x-1)}{2x-1} \geq 0$$

$$\frac{x-(2x-1)}{2x-1} \geq 0$$

$$\frac{-x+1}{2x-1} \geq 0$$

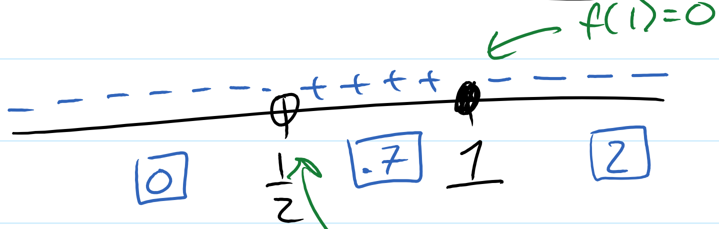
$-x+1=0$

$-x=-1$

$\boxed{x=1}$

$2x-1=0$

$\boxed{x=\frac{1}{2}}$



$x=\frac{1}{2}$ is excluded since $\frac{1}{2}$ is not in domain

$$f(x) = \frac{-x+1}{2x-1}$$

$$f(0) = \frac{0+1}{0-1} = -1 < 0$$

$$f(.7) = \frac{-0.7+1}{2\cdot 0.7-1} = \frac{0.3}{1.4-1} > 0$$

$$f(2) = \frac{-2+1}{2\cdot 2-1} = \frac{-1}{3} < 0$$

$$f(2) = \frac{-2+1}{2\cdot 2-1} = \frac{-1}{3} < 0$$

Solution: $\boxed{(\frac{1}{2}, 1]}$

Find domain:

$$f(x) = \sqrt{2 - \frac{x-3}{5+x}}$$

$$\frac{5+x}{5+x}\cdot 2 - \frac{x-3}{5+x} \geq 0$$

$$\frac{2(5+x)}{5+x} - \frac{x-3}{5+x} \geq 0$$

$$\frac{10+2x}{5+x} + \frac{-(x-3)}{5+x} \geq 0$$

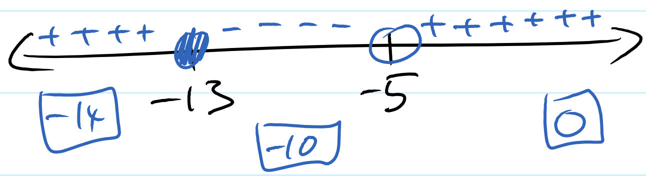
$$\frac{10+2x-(x-3)}{5+x} \geq 0$$

$$\frac{10+2x-x+3}{5+x} \geq 0$$

$$\frac{x+13}{5+x} \geq 0$$

$x+13=0$ | $5+x=0$
$x=-13$ | $x=-5$

$$f(x) = \frac{x+13}{5+x}$$



$$f(-14) = \frac{-1}{-9} = \frac{1}{9} > 0$$

$$\boxed{\text{Solution: } (-\infty, -13] \cup (-5, \infty)}$$

$$f(-10) = \frac{3}{-5} < 0$$

$$f(0) = \frac{13}{5} > 0$$