

Section 4.2

Ex: log. form exp. form

$$\log_3 9 = y \leftrightarrow 3^y = 9 \quad \boxed{y=2}$$

$$\log_3 9 = 2$$

$$\log_7 \frac{1}{49} = y \leftrightarrow 7^{\textcircled{y}} = \frac{1}{49} = \frac{1}{7^2} = 7^{\textcircled{-2}}$$

$$y = -2$$

$$\log_7 \frac{1}{49} = -2$$

$$\log_2 \sqrt[5]{2} = y \leftrightarrow 2^y = \sqrt[5]{2} = 2^{\frac{1}{5}}$$

$$\log_2 \sqrt[5]{2} = \frac{1}{5}$$

Properties of logarithm

1) $\log_b b = 1$, where $b > 0$ and $b \neq 1$

$$2) \log_b 1 = 0, \text{ where } b > 0 \text{ and } b \neq 1.$$

$$3) \left. \begin{array}{l} b^{\log_b x} = x \\ \log_b b^x = x \end{array} \right\} \text{ where } b > 0 \text{ and } b \neq 1.$$

Ex:

$$\log_4 4^5 = 5$$

$$6^{\log_6 9} = 9$$

Def: The natural logarithm is the log. function with base e , i.e.

$$\log_e x = \ln x$$

Def: The common logarithm is the log. function with base 10, i.e.

$$\log_{10} x = \log x$$

Ex: Evaluate:

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \dots$$

$$\bullet \log_2 32 = \log_2 2^5 = 5$$

$$\bullet \log_{25} 5 = \log_{25} (25)^{\frac{1}{2}} = \frac{1}{2}$$

Ex:

$$\bullet \ln e^{13x} = \log_e e^{13x} = 13x$$

$$\bullet \log(100 \cdot 10^x) = \cancel{(\log 100)} \cdot \log 10^x$$

$$= \log(10^2 \cdot 10^x) = \log(10^{2+x}) = \boxed{2+x}$$
$$= \log 10^2 + \log 10^x$$

Graphs

