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Monday, October 23, 2017 11:00 AM

Section 4.3

exponent form	log. form
$b^m \cdot b^n = b^{m+n}$	$\log_b(M \cdot N) = \log_b M + \log_b N$
$\frac{b^m}{b^n} = b^{m-n}$	$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

<u>Ex:</u> expand:

- $\log_4(7 \cdot 5) = \log_4 7 + \log_4 5$
- $\log(10x) = \log 10 + \log x = 1 + \log x$
- $\log_7\left(\frac{19}{x}\right) = \log_7 19 - \log_7 x$
- $\ln\left(\frac{e^3}{7}\right) = \ln e^3 - \ln 7 = 3 - \ln 7$

$$\boxed{\begin{aligned} e^{\ln x} &= x \\ \ln e^x &= x \end{aligned}}$$

exp	log
$(b^m)^n = b^{m \cdot n}$	$\log_b m^n = n \cdot \log_b m$

Ex: simplify:

- $\log_5 7^4 = 4 \cdot \log_5 7$
- $\ln \sqrt{x} = \ln x^{1/2} = \frac{1}{2} \ln x$
- $\log (4x)^5 = 5 \log 4x = 5(\log 4 + \log x)$
  $= 5 \log 4 + 5 \log x$
- $\log_2 (x^2-1)^4 = 4 \log_2 (x-1)(x+1)$
  $= \boxed{4 \log_2 (x-1) + 4 \log_2 (x+1)}$

$\neq \boxed{4 \log_2 (x^2-1)}$

$$= \boxed{4 \log_2 (x^2-1)}$$

Ex: Expand log. expression

- $\log_b (x^2\sqrt{y}) = \log_b x^2 + \log_b y^{1/2}$

$$= 2 \log_b x + \frac{1}{2} \log_b y$$

- $\log_6 \left(\frac{\sqrt[3]{x}}{36y^4}\right) = \log_6 x^{1/3} - \log_6 (36y^4)$

$$= \frac{1}{3} \log_6 x - [\log_6 36 + \log_6 y^4]$$

$$= \frac{1}{3} \log_6 x - \log_6 6^2 - \log_6 y^4$$

$$= \boxed{\frac{1}{3} \log_6 x - 2 - 4 \log_6 y}$$

Ex: Write as a single log.

- $\log_4 2 + \log_4 32 = \log_4 (2\cdot 32) = \log_4 64$

$= \log_4 4^3 = 3 \cdot \log_4 4$

$= \boxed{3}$

$= \log \quad 8^2 = 2 \log \quad 8$

$= \log_4 4^3 = 3 \cdot \log_4 4$

$= \boxed{3}$

$= \log_4 8^2 = 2 \log_4 8$

$= 2 \cdot \log_4 (2 \cdot 4) = 2 (\log_4 2 + \log_4 4)$

$= 2 \cdot (\frac{1}{2} + 1) = 2 \cdot 1.5 = \boxed{3}$

- $\log (4x-3) - \log x + 1$

$= \log \left(\frac{4x-3}{x} \right) + 1 = \log \left(\frac{4x-3}{x} \right) + \log 10$

$= \log \left(10 \cdot \frac{4x-3}{x} \right) = \log \left(\frac{40x-30}{x} \right)$

- $\frac{1}{2} \log x + 4 \log (x-1) = \log \sqrt{x} + \log (x-1)^4$

$= \log (\sqrt{x} \cdot (x-1)^4)$

- $3 \ln (x+7) - \ln x = \ln (x+7)^3 - \ln x$

$= \ln \left(\frac{(x+7)^3}{x} \right)$

- $4 \log_6 x - 2 \log_6 6 - \frac{1}{2} \log_6 y$

$= \underbrace{\log_b x^4 - \log_b 36} - \log_b \sqrt{y}$

$= \log_b\left(\frac{x^4}{36}\right) - \log_b \sqrt{y} = \log_b\left(\frac{\frac{x^4}{36}}{\sqrt{y}}\right)$

$= \boxed{\log_b\left(\frac{x^4}{36\sqrt{y}}\right)}$

$= \log_b x^4 - (\log_b 36 + \log_b \sqrt{y})$

$= \log_b x^4 - \log_b(36\sqrt{y}) = \boxed{\log_b \frac{x^4}{36\sqrt{y}}}$

Exp	log
$\frac{2}{2} = 1$	change of base formula
$2^x = \left(\frac{2\cdot 3}{3}\right)^x = \frac{(2\cdot 3)^x}{3^x}$	$\log_b M = \frac{\log_a M}{\log_a b}$

Ex: Simplify $\log_6 9$

$\log_6 9 = \frac{\log_3 9}{\log_3 6} = \frac{2}{\log_3(2\cdot 3)} = \frac{2}{\log_3 2 + \log_3 3}$

$$\log_6 9 = \frac{\log_3 9}{\log_3 6} = \frac{2}{\log_3(2\cdot 3)} = \frac{2}{\log_3 2 + \log_3 3}$$

$$= \boxed{\frac{2}{1+\log_3 2}}$$

Ex: Given: $\log 140 \approx 2.1$

$\log 5 \approx 0.7$

approximate:

$$\log_5 140 = \frac{\log_{10} 140}{\log_{10} 5} \approx \frac{2.1}{0.7} = \frac{21}{7} = \boxed{3}$$

Ex: Rewrite as a single log:

$$\frac{1}{3}\left(\log_4 x - \log_4 y\right) + 2\log_4(x+1)$$

$$= \frac{1}{3}\log_4 x - \frac{1}{3}\log_4 y + \log_4 (x+1)^2$$

$$= \log_4 \sqrt[3]{x} - \log_4 \sqrt[3]{y} + \log_4 (x+1)^2$$

$$= \boxed{\log_4\left(\frac{\sqrt[3]{x}(x+1)^2}{\sqrt[3]{y}}\right)}$$