10/27

Friday, October 27, 2017 11:02 AM

Section 4.4

Ex: solve:

$$\frac{3e^{1-8x}}{3} = \frac{7957}{3}$$

$$e^{1-8x} = \frac{7957}{3}$$

$$\ln e^{1-8x} = \ln\left(\frac{7957}{3}\right)$$

$$(1-8x)\ln e = \ln\left(\frac{7957}{3}\right)$$

$$1-8x = \ln\left(\frac{7957}{3}\right)$$

$$-1 \qquad\qquad -1$$

$$\frac{-8x}{-8} = \frac{\ln\left(\frac{7957}{3}\right) - 1}{-8}$$

$$x = \boxed{\frac{1}{8} - \frac{1}{8}\ln\left(\frac{7957}{3}\right)} \approx -0.86034$$

$$3^{2x} + 3^x - 2 = 0$$

$$\left(3^x\right)^2 + \left(3^x\right) - 2 = 0$$

substitute $z = 3^x$

$(3^x)^2 + (3^x) - 2 = 0$

substitute: $u = 3^x$

$u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$u = -2$ | $u = 1$

$3^x = -2$ | $3^x = 1$

$x \log_3 3 = \log_3 -2$ | $x = \log_3 1$

$x = \log_3 -2$ | $\boxed{x = 0}$

none

$3^x = 9^{x-1} \cdot 27^{1-3x}$

$3^x = (3^2)^{x-1} \cdot (3^3)^{1-3x}$

$3^x = 3^{2x-2} \cdot 3^{3-9x}$

$3^x = 3^{2x-2+3-9x}$

$3^x = 3^{-7x+1}$ $\longrightarrow$ $x = -7x + 1$

$8x = 1$

$\boxed{x = 1/8}$

Logarithmic Equations

$\log_2 9 = y$

$\log_3 9 = y$

$\Updownarrow$

$3^y = 9$

Solve: $\log_4 (x+3) = 2$

$\Updownarrow$

$4^2 = x+3$

$16 = x+3$

$x = 13$

check: $\log_4 (13+3) \stackrel{?}{=} 2$

$\log_4 16 = 2$ ✓

$\boxed{x = 13}$

- $3\ln(2x) = 12$

$\ln(2x) = 4 \longrightarrow e^4 = 2x$

$\boxed{x = \frac{1}{2}e^4}$

check: $3\ln\left(2 \cdot \frac{1}{2}e^4\right) \stackrel{?}{=} 12$

$3 \cdot \ln(e^4) = 12$

$3 \cdot 4 = 12$ ✓

$x = \frac{1}{2} e^4$

- $\log_2 x + \log_2 (x-7) = 3$

$$\log_2 [x(x-7)] = 3$$

$$2^3 = x(x-7)$$

$$8 = x^2 - 7x$$

$$0 = x^2 - 7x - 8$$

$$0 = (x-8)(x+1)$$

$$x = 8, \quad x = -1$$

check: $x = 8$

$$\log_2 8 + \log_2 (8-7) = 3$$

$$3 + \log_2 1 = 3$$

$$3 + 0 = 3 \quad \checkmark$$

$x = -1$

$$\log_2 (-1) + \log_2 (-1-7) = 3$$

no

$x = 8$

- $\ln(x+2) - \ln(4x+3) = \ln\left(\frac{1}{x}\right)$

$$\ln\left(\frac{x+2}{4x+3}\right) = \ln\left(\frac{1}{x}\right)$$

$$\downarrow$$

$$\frac{x+2}{4x+3} = \frac{1}{x}$$

$x \neq 0, -\frac{3}{4}$ since $\frac{1}{0}$ is not defined

$$x(x+2) = 1 \cdot (4x+3)$$

$$x^2 + 2x = 4x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$x = 3$

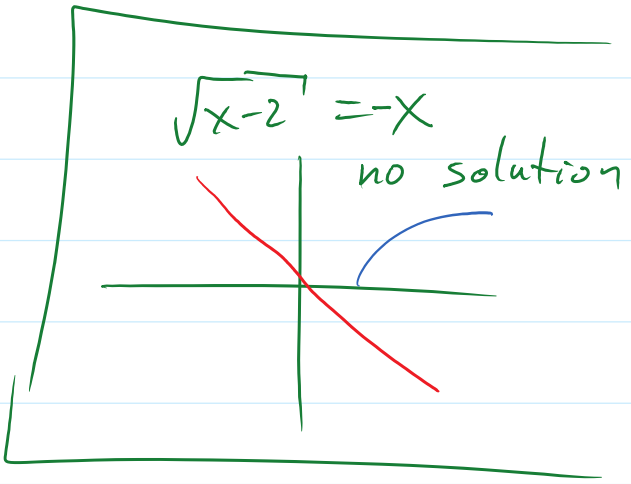
$$\ln(3+2) - \ln(4 \cdot 3 + 3) = \ln\left(\frac{1}{3}\right)$$

$$\ln(5) - \ln(15) = \ln\left(\frac{1}{3}\right)$$

$x = -1$

$$\ln(-1+2) - \ln(4 \cdot (-1) + 3) = \ln\left(\frac{1}{-1}\right)$$

$$\ln(1) - \ln(-1) = \ln(-1)$$



• $\ln(x-3) = \ln(7x-23) - \ln(x+1)$

$$\ln(x-3) = \ln\left(\frac{7x-23}{x+1}\right)$$

$$\downarrow$$

$$x-3 = \frac{7x-23}{x+1}$$

$$(x-3)(x+1) = 7x-23$$
$$x^2 - 2x - 3 = 7x - 23$$
$$x^2 - 9x + 20 = 0$$
$$(x-4)(x-5) = 0$$

$x = 4$

$\ln(4-3) = \ln(7\cdot 4 - 23) - \ln(4+1)$

$\ln(1) = \ln(5) - \ln(5)$ ✓

$x = 5$

$\ln(5-3) = \ln(7\cdot 5 - 23) - \ln(5+1)$

$\ln 2 = \ln(12) - \ln 6$ ✓

$\boxed{x = 4, 5}$

- $\log(3x-3) = \log(x+1) + \log 4$

$\log(3x-3) = \log(4(x+1))$

$\downarrow$

$3x-3 = 4(x+1)$

$3x-3 = 4x+4$

$x = -7$

$\log(3(-7)-3) = \log(-7+1) + \log 4$

negative negative

no solution, the solution set $= \emptyset$