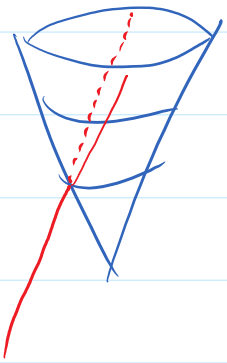
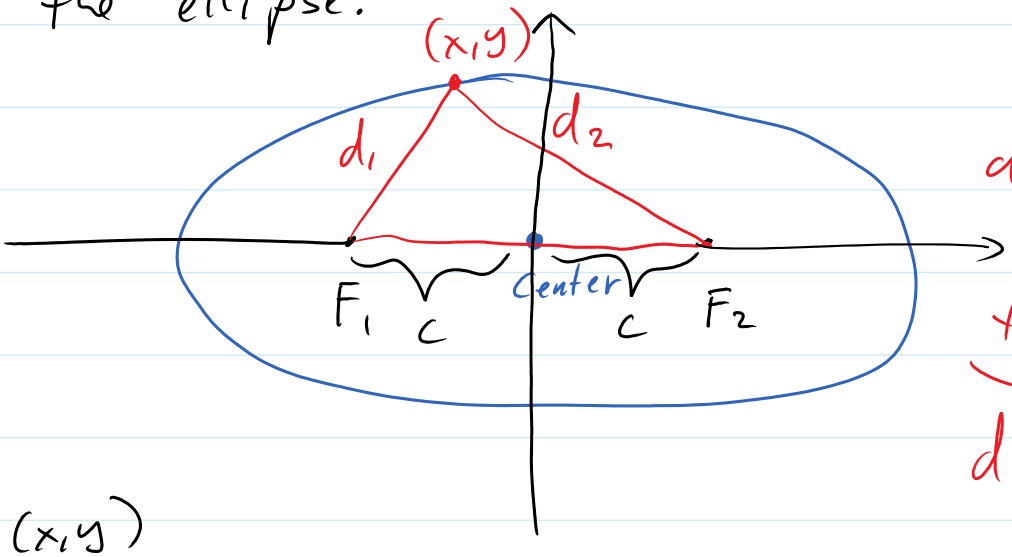
11/01

Wednesday, November 1, 2017 8:40 AM



Section 10.1

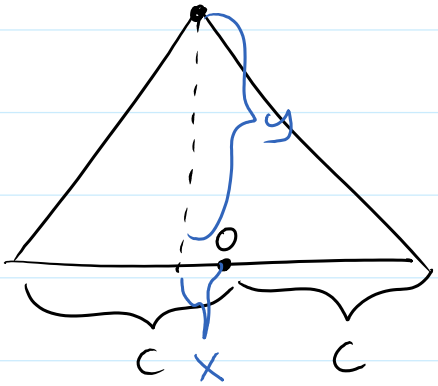
Def: An ellipse is the set of all points, P, in a plane the sum of whose distances from two fixed points, F_1, F_2, is constant. These two fixed points are called foci. The midpoint of the segment connecting the foci is the center of the ellipse.



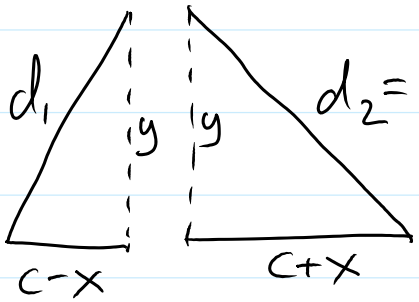
d_1+d_2 is constant for all (x,y)

$d_1+d_2=2a$

(x,y)



$\sqrt{y^2+(c-x)^2} = d_1$

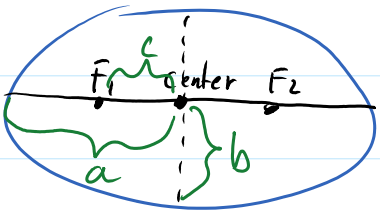


$d_2 = \sqrt{y^2+(c+x)^2}$

$$d_1 + d_2 = 2a$$

$$\sqrt{y^2+(c-x)^2} + \sqrt{y^2+(c+x)^2} = 2a$$

$$\vdots$$



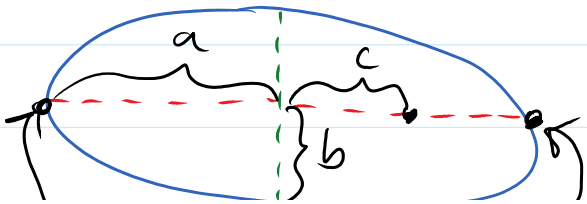
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$a^2 = c^2 + b^2$$

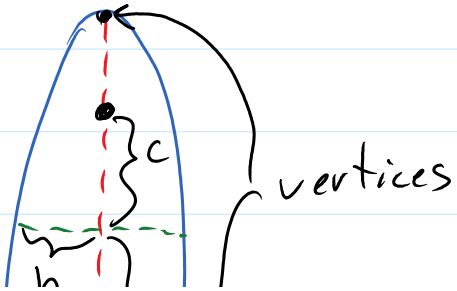
$$c^2 = a^2 - b^2$$

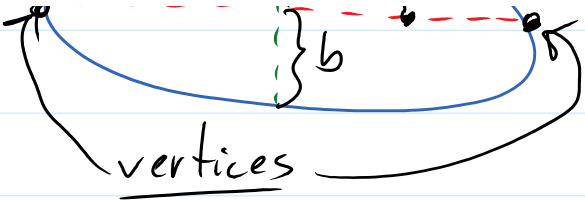
Standard eq. of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

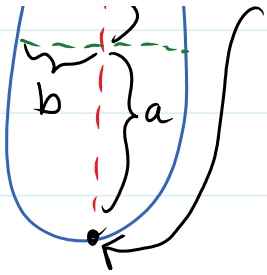


$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$





horizontal major axis



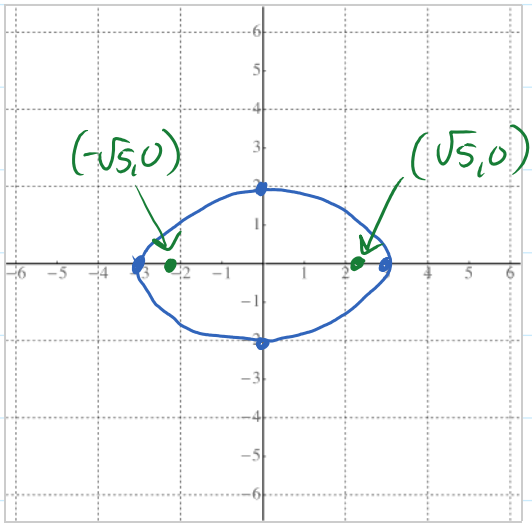
vertical major axis

Ex: Graph and locate foci: $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$a^2 = 9$

$b^2 = 4$

horizontal major axis



$c^2 = a^2 - b^2$

$c^2 = 9 - 4 = 5$

$c = \sqrt{5}$

Ex:

$$\frac{25x^2}{400} + \frac{16y^2}{400} = \frac{400}{400}$$

$$\frac{25x^2}{400} + \frac{16y^2}{400} = 1$$

$$\frac{25x^2}{400} + \frac{16y^2}{400} = 1$$

$$\frac{x^2}{\frac{400}{25}} + \frac{y^2}{\frac{400}{16}} = 1$$

$$\frac{x^2}{\left(\frac{20}{5}\right)^2} + \frac{y^2}{\left(\frac{20}{4}\right)^2} = 1$$

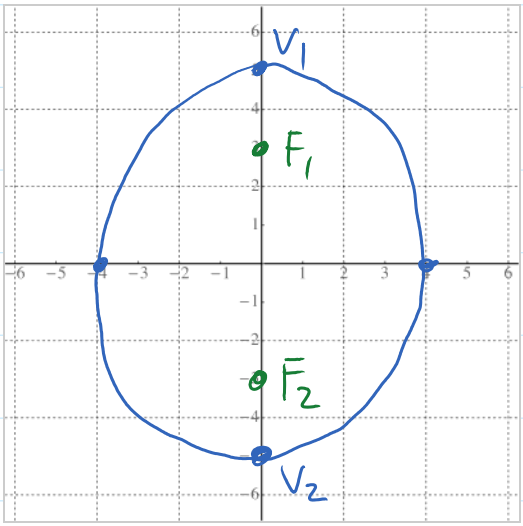
$$\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$$

major axis vertical: $a=5, b=4$

$c^2 = 5^2 - 4^2$

$c^2 = 25 - 16 = 9$

$c = 3$



Find the standard form of the equation of an ellipse with foci $(\pm 1, 0)$ and vertices $(\pm 2, 0)$

vertices $(\pm 2, 0)$ → horiz. maj axis.

$c=1,\ a=2 \Rightarrow c^2 = a^2 - b^2$

$1 = 4 - b^2$

$b^2 = 3 \rightarrow b = \sqrt{3}$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$