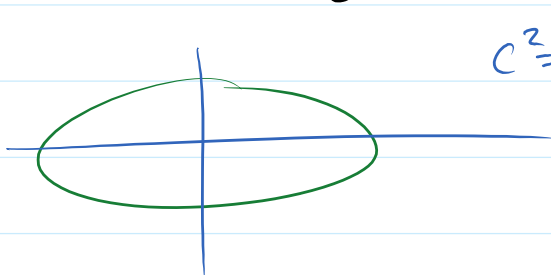
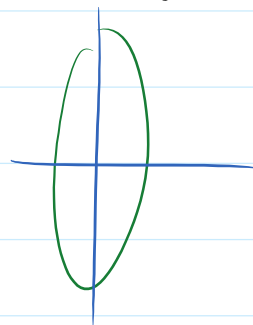


## Section 10.1 - Ellipse

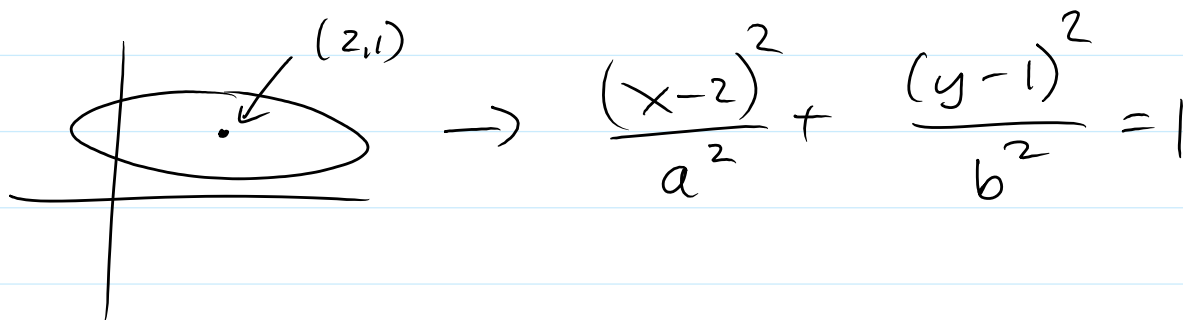
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



$$c^2 = a^2 - b^2$$



Can we shift the ellipse?



The standard form of the ellipse with center at  $(h, k)$  is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{OR} \quad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Ex: Graph and locate the foci

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

$$\frac{(x-1)}{4} + \frac{(y+2)}{9} = 1$$

• Vertical maj. axis

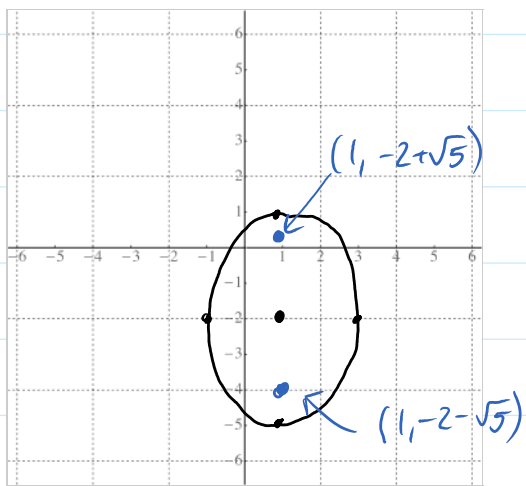
$$a^2 = 9, \quad b^2 = 4$$

center:  $(1, -2)$

$$c^2 = a^2 - b^2$$

$$c^2 = 9 - 4 = 5$$

$$c = \sqrt{5}$$



foci:  $(1, -2 \pm \sqrt{5})$

locate the center, vertices and foci:

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$

get it  
to the  
stand. form

$$9x^2 - 18x + 4y^2 + 16y = 11$$

$$9(x^2 - 2x) + 4(y^2 + 4y) = 11$$

$$\frac{-2}{2} = -1 + (-1)^2$$

$$\frac{4}{2} = 2 + (2)^2$$

complete  
the square

$$9(x^2 - 2x + (-1)^2) + 4(y^2 + 4y + 2^2) = 11 + 9 + 4 \cdot 4$$

$$\frac{9(x-1)^2}{36} + \frac{4(y+2)^2}{36} = \frac{36}{36}$$

$$\frac{(x-1)^2}{\frac{36}{9}} + \frac{(y+2)^2}{\frac{36}{4}} = 1$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

And this is the same as the prev. problem ☺.

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Ex:  $4x^2 + 36y^2 + 40x - 288y + 532 = 0$

$$4x^2 + 40x + 36y^2 - 288y = -532$$

$$4(x^2 + 10x + 5^2) + 36(y^2 - 8y + (-4)^2) = -532 + 4 \cdot 5^2 + 36 \cdot (-4)^2$$

$\frac{10}{2} = 5$                        $\frac{-8}{2} = -4$

$$\frac{4(x+5)^2}{144} + \frac{36(y-4)^2}{144} = -532 + 100 + 36 \cdot 16$$

$$= \frac{144}{144}$$

$$\frac{(x+5)^2}{36} + \frac{(y-4)^2}{4} = 1$$

• center:  $(-5, 4)$

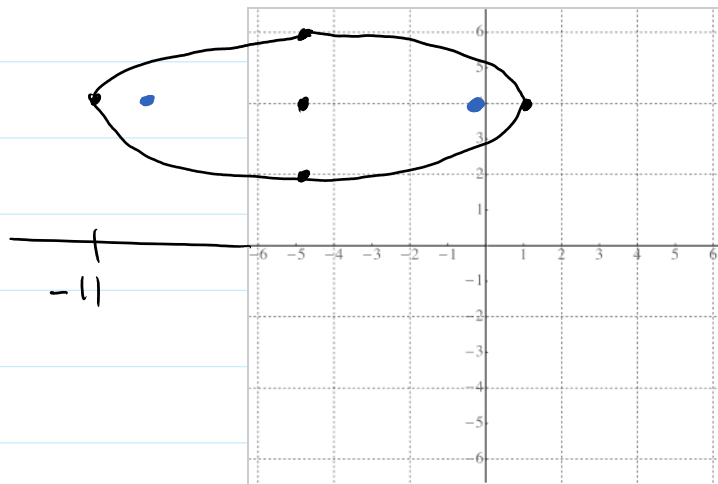
- $a^2 = 36$ ,  $b^2 = 4$
- major axis horizontal
- $c^2 = a^2 - b^2$

$$c^2 = 36 - 4 = 32$$

$$c = \sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$$

• vertices:  $(-5 \pm 6, 4) = \boxed{(-11, 4), (1, 4)}$

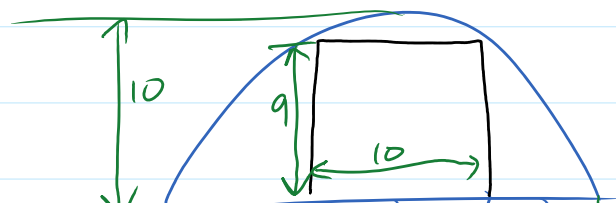
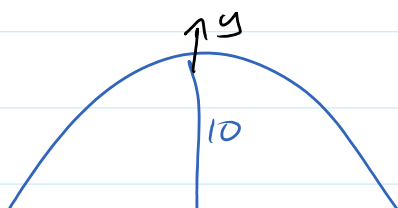
• foci:  $(-5 \pm 4\sqrt{2}, 4)$

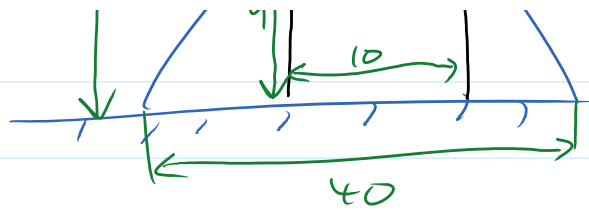
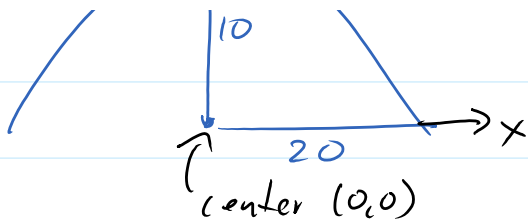


Ex: A semielliptical archway over a one-way road has a height 10 feet and a width of 40 feet.

Your truck has a width of 10 feet and a height of 9 feet.

Can you fit under this arch?





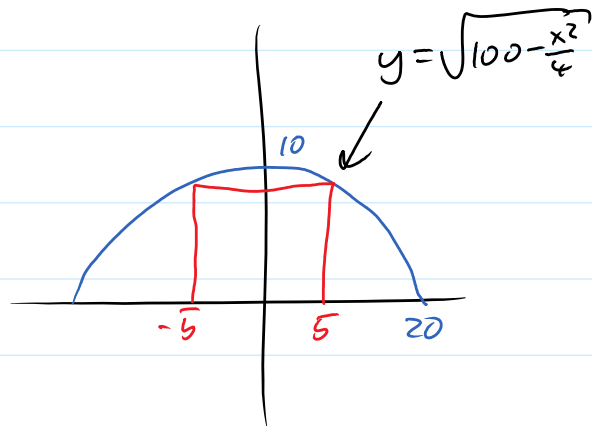
$$\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1$$

solve for  $y$

$$\frac{y^2}{10^2} = 1 - \frac{x^2}{20^2}$$

$$y^2 = 10^2 - \frac{100x^2}{400}$$

$$y = \sqrt{100 - \frac{x^2}{4}}$$



$y(5) = \dots$  the height of the arch when  $x=5$

$$y(5) = \sqrt{100 - \frac{25}{4}} \approx \boxed{9.7}$$

Since  $9 < 9.7$  our truck will fit.