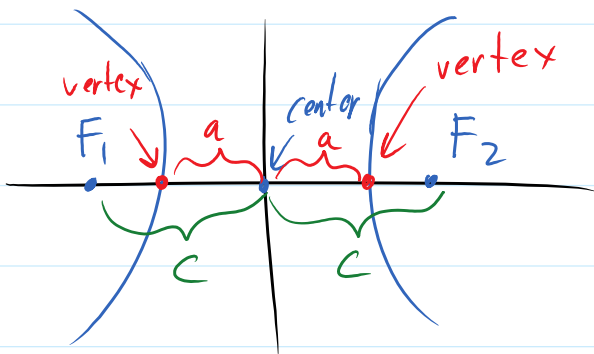


Section 10.2

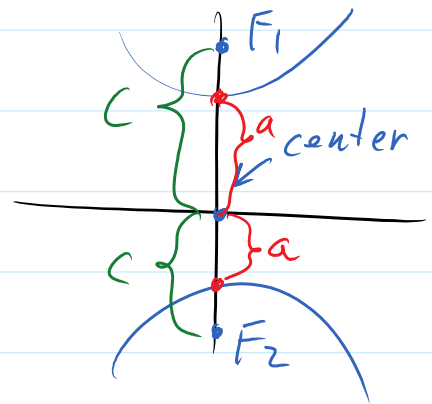
Def: A hyperbola is the set of points in a plane the difference of whose distances from two fixed points, called foci, is constant.

Standard Forms of the Equations of a Hyper

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



$$c^2 = a^2 + b^2$$

Ex: Find the vertices and foci for each of the following hyperbolas:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

• hor. trans. axis

$$a^2 = 16 \rightarrow a = 4$$

$$b^2 = 9 \rightarrow b = 3$$

$$c^2 = a^2 + b^2$$

$$c^2 = 16 + 9 = 25$$

$$c = 5$$

• Foci: $(\pm 5, 0)$

• vertices: $(\pm 4, 0)$

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

• vertical trans. axis

$$a^2 = 9 \rightarrow a = 3$$

$$b^2 = 16 \rightarrow b = 4$$

$$c^2 = 9 + 16 = 25$$

$$c = 5$$

• Foci: $(0, \pm 5)$

• Vertices: $(0, \pm 3)$

Ex: Find the standard form of the equation of a hyperbola with foci at $(0, \pm 3)$ and vertices at $(0, \pm 2)$.

$$a = 2$$

$$c = 3$$

$$c^2 = a^2 + b^2$$

$$9 = 4 + b^2$$

$$b^2 = 5$$

$$b = \sqrt{5}$$

vertical trans.
axis.

$$\boxed{\frac{y^2}{4} - \frac{x^2}{5} = 1}$$

Asymptotes

The hyperbola $\left\{ \begin{array}{l} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \end{array} \right\}$ has two asymptotes

$$y = \frac{b}{a}x \quad \& \quad y = -\frac{b}{a}x$$

$$y = \frac{a}{b}x \quad \& \quad y = -\frac{a}{b}x$$

Graph and locate foci:

$$\frac{x^2}{25} - \frac{y^2}{16} = 1$$

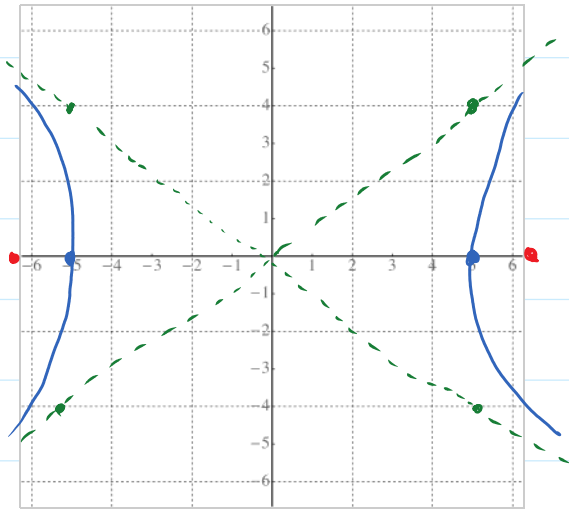
• horizontal trans. axis

$$\left. \begin{array}{l} \bullet a^2 = 25 \rightarrow a = 5 \\ \bullet b^2 = 16 \rightarrow b = 4 \end{array} \right\}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 25 + 16 = 41 \end{aligned}$$

$$c = \sqrt{41}$$

• asym: $y = \pm \frac{4}{5}x$



Find the std. eq.

$$\frac{9y^2}{36} - \frac{4x^2}{36} = \frac{36}{36}$$

$$\frac{y^2}{\frac{36}{9}} - \frac{x^2}{\frac{36}{4}} = 1$$

$$\boxed{\frac{y^2}{4} - \frac{x^2}{9} = 1}$$

$$\left. \begin{array}{l} a^2 = 4 \\ b^2 = 9 \end{array} \right\} \text{asym: } y = \pm \frac{2}{3}x$$

$c = \sqrt{13}$
• vertical trans. axis