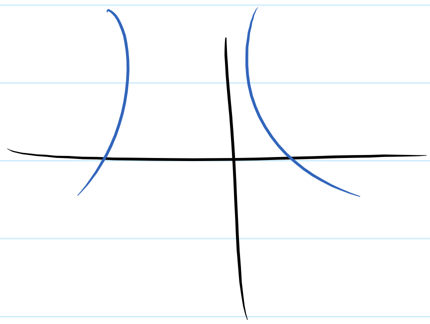
11/08

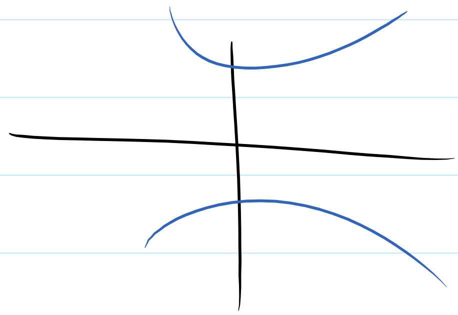
Wednesday, November 8, 2017 11:02 AM

Section 10.2

Standard eq.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$





Ex: graph $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{9} = 1$

$a^2 = 16 \to a = 4$

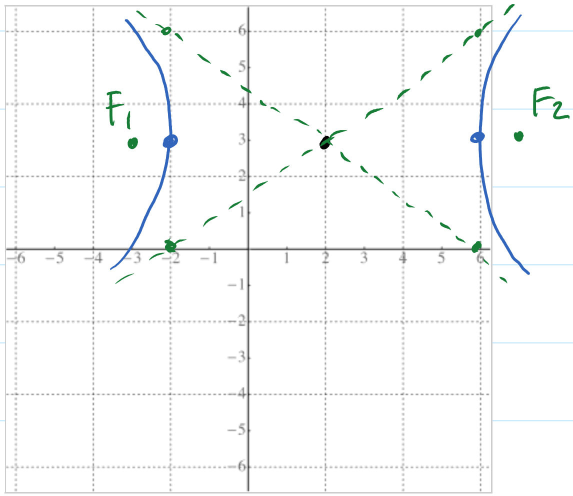
$b^2 = 9 \to b = 3$

$c^2 = a^2 + b^2$

$c^2 = 16 + 9 = 25$

$c = 5$

- center: $(2,3)$
- foci: $(2 \pm 5, 3) = (7,3), (-3,3)$
- vertices: $(2 \pm 4, 3) = (6,3), (-2,3)$
- asymp: $\boxed{y = \pm \frac{3}{4}(x-2) + 3}$



Graph: $4x^2 - 24x - 25y^2 + 250y - 489 = 0$

$$4(x^2 - 6x + (-3)^2) - 25(y^2 - 10y + (-5)^2) = 489$$

$$+4\cdot 9 \quad -25\cdot 25$$

$$4(x-3)^2 - 25(y-5)^2 = 489 + 36 - 625$$

$$= 525 - 625$$

$$\frac{4(x-3)^2}{-100} - \frac{25(y-5)^2}{-100} = \frac{-100}{-100}$$

$$\frac{-(x-3)^2}{25} + \frac{(y-5)^2}{4} = 1$$

$$\boxed{\frac{(y-5)^2}{2^2} - \frac{(x-3)^2}{5^2} = 1}$$

vertical trans. axis

$a = 2 \}$ $\quad c^2 = a^2 + b^2$

$$\left.\begin{array}{l} a=2 \\ b=5 \end{array}\right\} \quad \begin{array}{l} c^2=a^2+b^2 \\ c^2=4+25=29 \\ c=\sqrt{29} \end{array}$$

- center: $(3, 5)$
- foci: $(3, 5 \pm \sqrt{29})$
- vert: $(3, 5 \pm 2)$
- asym: $\boxed{y = \pm \frac{2}{5}(x-3)+5}$

Ex: Find the standard eq. of the hyperbola with center at $(4,-2)$ focus $(7,-2)$, vertex: $(6,-2)$

horizontal trans. axis.

$a = 6-4 = 2$

$b = ?$

$c = 7-4 = 3$

$c^2 = a^2+b^2$

$9 = 4+b^2$

$b^2 = 5 \rightarrow b = \sqrt{5}$

$$\boxed{\frac{(x-4)^2}{4} - \frac{(y+2)^2}{5} = 1}$$

asymptote: $\boxed{y = \pm \frac{\sqrt{5}}{2}(x-4) - 2}$

Find the st. form of the eq. of hyperbola:

asymptote: $y = 2x$

vertices: $(-4, 0), (4, 0)$

- center: $(0,0)$
- $a = 4$
- hor. trans. axis

$$y = \frac{b}{a} x$$

$$\frac{b}{a} = 2 \longrightarrow \frac{b}{4} = 2$$

$$\boxed{b = 8}$$

$$c^2 = a^2 + b^2$$

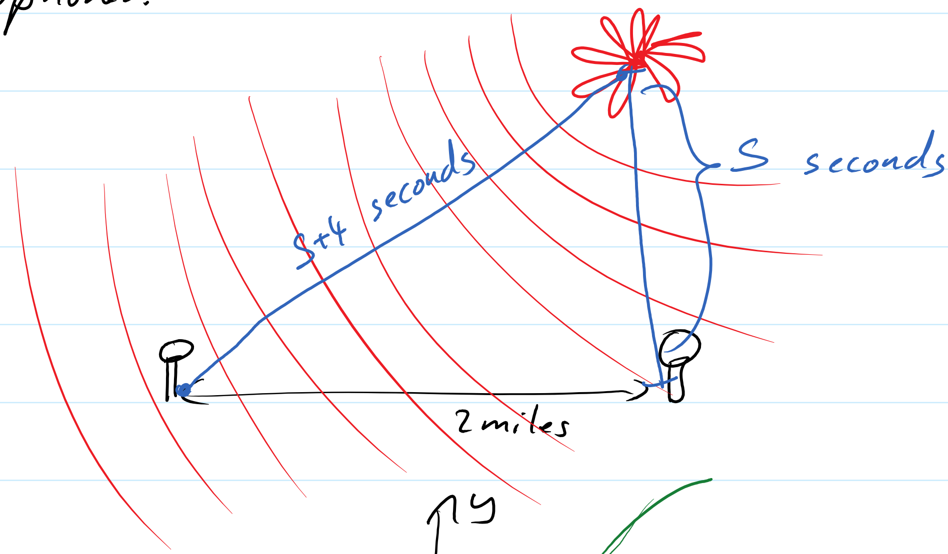
$$c^2 = 16 + 64 = 80$$

$$c = \sqrt{80} = 2\sqrt{20} = 4\sqrt{5}$$

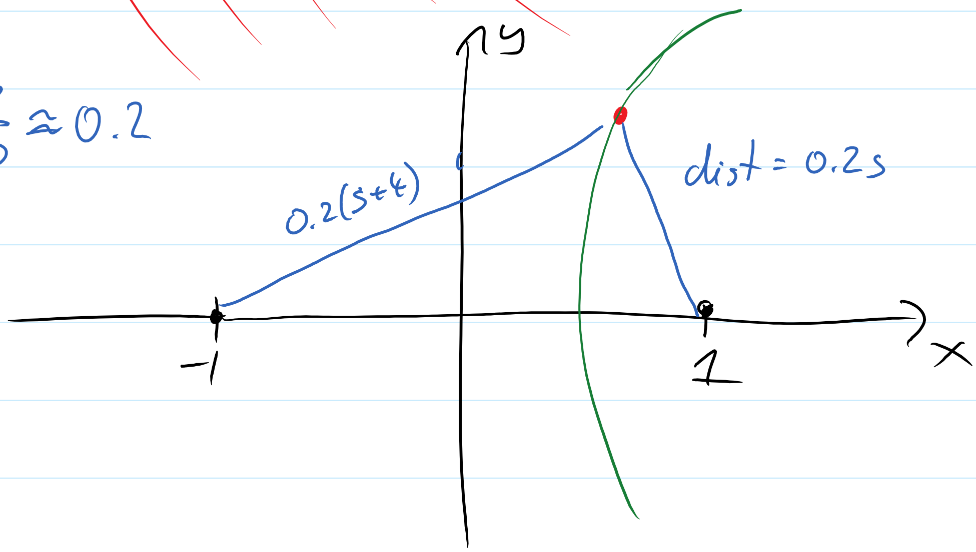
$$\boxed{\frac{x^2}{16} - \frac{y^2}{64} = 1} \qquad \boxed{\text{foci: } (\pm 4\sqrt{5}, 0)}$$

Ex: An explosion is recorded by two microphones that are 2 miles apart. Microphone M_1 received the sound 4 seconds before microphone M_2. Assuming that sound travels 1100 feet per second, determine the position of the explosion relative to location of the microphones.

of the explosion relative to location of the microphones.



$$1100\text{ ft} = \frac{1100}{5280} \approx 0.2$$



since the set of points will create a hyperbola, by definition 2a = "the difference of distances"

we get

$$2a = 0.2(s+4) - 0.2s$$

$$2a = 0.2s + 0.8 - 0.2s$$

$$2a = 0.8$$

$$\underline{a = 0.4}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \longrightarrow \frac{x^2}{(0.4)^2} - \frac{y^2}{b^2} = 1$$

Since $c =$ "distance between center $(0,0)$ and a vertex"
$(\pm 1, 0)$

$\underline{c = 1}$

$c^2 = a^2 + b^2$ let's find $\underline{b}$

$1 = (0.4)^2 + b^2$

$b^2 = 1 - 0.16$

$b^2 = .84$

$b \approx .9$

The equation of all possible positions for the explosion is

$$\frac{x^2}{0.16} - \frac{y^2}{0.84} = 1$$

We cannot determine the location exactly, but we can say that the explosion was located somewhere on the hyperbola.