Exam review on Saturday 11/18, 10am-noon at MMC, CP145

Online Office hour on Sunday, 11/19, starting at 8pm

Section 10.3 $(x-h)^2 = 4p(y-k)$

Ex: Find the vertex, focus and directrix of the parabola

$$(x-3)^2 = 8(y+1)$$

$4p$

vertex: $\boxed{(3, -1)}$

focus: $(3, -1+2) = \boxed{(3, 1)}$
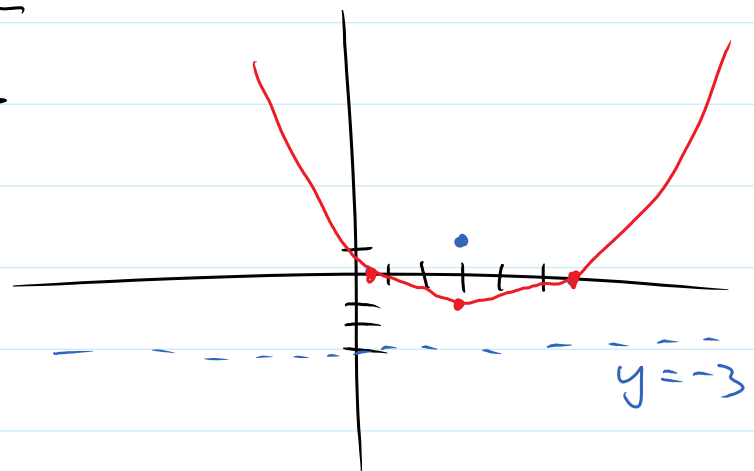
$\dfrac{8}{4} = \dfrac{4p}{4}$

$p = 2$

directrix: $y = -1 - 2$

$\boxed{y = -3}$

x-int: $(x-3)^2 = 8(y+1)$

$(x-3)^2 = 8(0+1)$

$(x-3)^2 = 8$

$y = -3$

$$(x-3)^2 = 8$$
$$x - 3 = \pm\sqrt{8}$$
$$x = 3 \pm 2\sqrt{2}$$
$$x = 5.8, \ 0.17$$

Find vertex, focus and directrixot

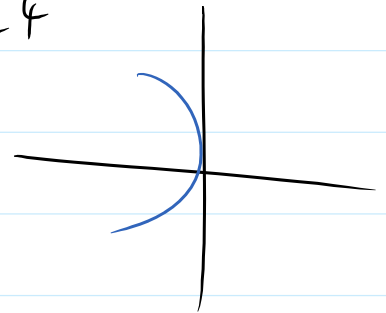$$y^2 + 2y + 12x - 23 = 0$$

$$y^2 + 2y + 1^2 = -12x + 23 + 1^2$$
$$(y+1)^2 = -12x + 24$$
$$(y+1)^2 = -12(x - 2)$$

$\underbrace{}_{4p}$

$$4p = -12$$
$$p = -3$$

vertex: $\boxed{(2, -1)}$

focus: $(2-3, -1)$
$$= \boxed{(-1, -1)}$$
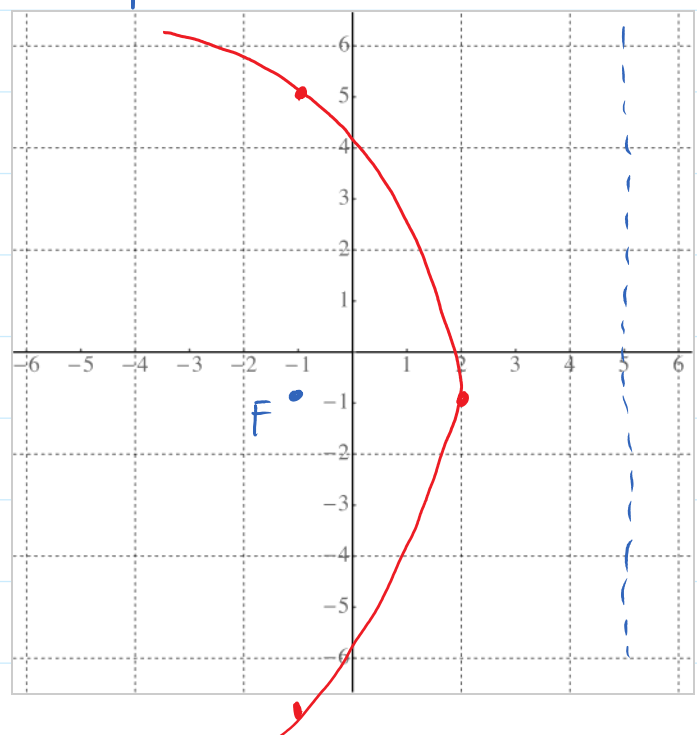
direct: $x = 2 - (-3)$
$$\boxed{x = 5}$$

$$(y+1)^2 = -12(x-2)$$

$\underline{x = -1:}$
$$(y+1)^2 = -12(-3)$$
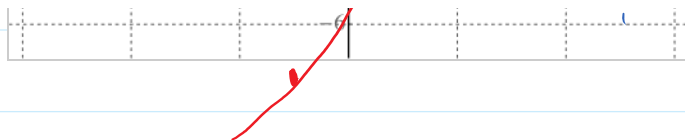$$(y+1)^2 = +36$$
$$y+1 = \pm 6$$

$$(y+1) = \pm 6$$
$$y + 1 = \pm 6$$
$$y = \pm 6 - 1$$
$$(-1, 5), \quad (-1, -7)$$

Ex: Find vertex, focus and direct.

$$x^2 + 2x + 4y - 7 = 0$$
$$x^2 + 2x + 1^2 = -4y + 7 + 1^2$$
$$(x+1)^2 = -4(y-2)$$
$$\underset{4p}{\underbrace{\quad}}$$

par. opens down

vertex: $\boxed{(-1, 2)}$

focus: $(-1, 2-1)$
$$= \boxed{(-1, 1)}$$

direct: $y = 2 - (-1)$
$$\boxed{y = 3}$$

$$4p = -4$$
$$\underline{p = -1}$$

points: $y = 1$
$$(x+1)^2 = -4(1-2)$$
$$(x+1)^2 = 4$$
$$x + 1 = \pm 2$$
$$x = \pm 2 - 1$$
$$\boxed{(1,1), \quad (-3, 1)}$$

Classification of Conic sections

Ex: classify:

- $3x^2 + 4y^2 - 2y = 100$ ⟩ complete the square

both positive and x^2, y^2

we get an ellipse

$$3x^2 + 4\left(y^2 - \tfrac{1}{2}y + \left(\tfrac{-1}{4}\right)^2\right) = 100 + 4 \cdot \left(\tfrac{-1}{4}\right)^2$$

$$3x^2 + 4\left(y - \tfrac{1}{4}\right)^2 = 100 + \tfrac{1}{4}$$

- $4x^2 - 25y^2 - 4x + 250y - 489 = 0$

→ either ellipse or hyperbola

but

→ differ in sign ⟹ it's a hyperbola

Rule:

A conic section given by
$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

is

- a circle if $A = C$
- a parabola if $A \cdot C = 0$
  - either $A = 0$ or $C = 0$
- an ellipse if $A \neq C$ and $\underline{A \cdot C > 0}$

- an ellipse if $A \neq C$ and $A \cdot C > 0$

A and C have the same sign

- a hyperbola if $A \cdot C < 0$

$\hookrightarrow A$ and C have different sign

Identify conic sections:

- $x^2 + y^2 + 6x - 2y + 6 = 0$

A circle

- $y^2 + 12x + 2y - 23 = 0$

A parabola (opens $> \subset$)

- $-3x^2 - 3y^2 + 4x - 5y + 10 = 0$

A circle.

- $9x^2 + 25y^2 - 54x - 50y - 119 = 0$

An ellipse.