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Wednesday, November 22, 2017 9:36 AM

Ex: Evaluate:

$$6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$$

$$0! = 1$$

$$(-1)! \quad \text{DNE}$$

Simplify:

$$\frac{(n+1)!}{n!} = \frac{(n+1)\overbrace{\cancel{n}\cdot\cancel{(n-1)}\,\cancel{(n-2)}\cdots\cancel{2}\cdot\cancel{1}}^{n!}}{\cancel{n}\cdot\cancel{(n-1)}\,\cancel{(n-2)}\cdots\cancel{2}\cdot\cancel{1}} = \frac{n+1}{1} = n+1$$

OR

$$= \frac{(n+1)\,\overbrace{\cancel{(n!)}}}{\cancel{n!}} = \boxed{n+1}$$

$$\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)\overbrace{(n)\,(n-1)\cdots2\cdot1}^{\cancel{n!}}}{\cancel{n!}} = \frac{(n+2)(n+1)}{1}$$

$$= \boxed{n^2+3n+2}$$

Sum notation

The sum of the first n terms of a sequence is represented by the summation notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_n,$$

where i (k) is the index of summation, n is the upper limit of summation, and 1 is the lower limit of summation.

Ex: Expand and evaluate:

$$\sum_{i=1}^{6} (i^2+1) = (1^2+1) + (2^2+1) + (3^2+1) + (4^2+1) + (5^2+1) + (6^2+1)$$

$$= 6 + 1 + 4 + 9 + 16 + 25 + 36 = \boxed{97}$$

$$\sum_{i=4}^{7} \left[(-2)^i - 5\right] = (-2)^4 - 5 + (-2)^5 - 5 + (-2)^6 - 5 + (-2)^7 - 5$$

$$= -20 + 16 - 32 + 64 - 128$$

$$= -20 - 16 - 64 = \boxed{-100}$$

$$\sum_{k=2}^{5}(3) = 3+3+3+3 = \boxed{12}$$

Find the genaral term of the sequence (a_n)

- $-2, 4, -8, 16, \ldots$

$(-2)^1, (-2)^2, (-2)^3, (-2)^4, (-2)^5, \ldots$

$a_1, a_2, a_3, \ldots$

$$a_n = (-2)^n$$

- $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}$

+1 +1

+2 +2

$$\boxed{a_n = \frac{n}{2n+1}}$$

$\frac{1}{3}$	$\frac{2}{5}$	$\frac{3}{7}$	
$\frac{1}{2\cdot1+1}$	$\frac{2}{2\cdot2+1}$	$\frac{3}{2\cdot3+1}$	$\ldots$

- $-3, 2, 7, 12, 17, \ldots$

+5 +5 +5

$$\boxed{a_n = 5n-8}$$

$5\cdot1-8, 5\cdot2-8, 5\cdot3-8, 5\cdot4-8, 5\cdot5-8$

Ex: Rewrite using the sum notation:

- $-3+2+7+12+17+\dots+52$

$$=\boxed{\sum_{i=1}^{12} 5i-8}$$

Find the upper lim

$52=5i-8$

$60=5i$

$i=12$

- $\frac{3}{2} \overset{\times 3}{} - \frac{9}{3} + \frac{27}{4} - \frac{81}{5}$

+1

$$\frac{3^{1}(-1)^{1+1}}{1+1} \quad \frac{3^{2}(-1)^{2+1}}{2+1} \quad \frac{3^{3}(-1)^{3+1}}{3+1} \quad \frac{3^{4}(-1)^{4+1}}{4+1}$$

gen. form:

$$a_n=\frac{3^n(-1)^{n+1}}{n+1}$$

$$=\boxed{\sum_{i=1}^{4} \frac{3^i(-1)^{i+1}}{i+1}} = \sum_{i=1}^{4} \frac{-3^i(-1)^i}{i+1}$$

$$=\sum_{i=1}^{4} -\frac{(-3)^i}{i+1}$$

Properties of Sums

1) $\sum_{i=1}^{n} c\cdot a_i = c\cdot \sum_{i=1}^{n} a_i$

2) $\sum_{i=1}^{n}(a_i+b_i) = \left(\sum_{i=1}^{n} a_i\right) + \left(\sum_{i=1}^{n} b_i\right)$

$$\sum_{i=1}^{10}(3+i) = \sum_{i=1}^{10} 3 + \sum_{i=1}^{10} i$$

3) $\sum_{i=1}^{n}(a_i-b_i) = \left(\sum_{i=1}^{n} a_i\right) - \left(\sum_{i=1}^{n} b_i\right)$

$$\sum a_i b_i \neq \sum a_i \cdot \sum b_i$$

NO