11/27

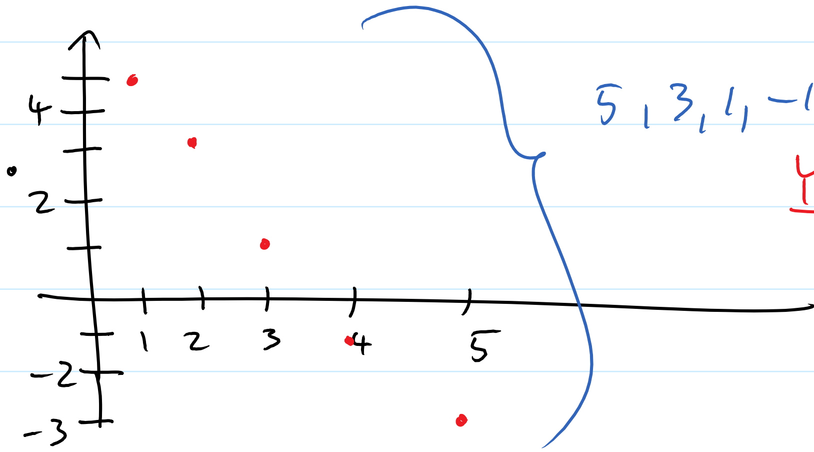
Monday, November 27, 2017 10:58 AM

Section 11.2

Def: An arithmetic sequence is a sequence in which each term after the first differs from the preceding term by a constant amount. The difference between consecutive terms is called the common difference of the sequence.

Ex: Is it an arith. seq?

- 2, 4, 8, 16, 32, 64, … NO
  $\underbrace{4,8}_{4}$ $\underbrace{8,16}_{8}$
- 2, 0, 2, 0, 2, 0, …. NO
  -2, $+2$, -2, $+2$
- 2, 4, 6, 8, 10, 12, … Yes
  $+2$, $+2$, $+2$, $+2$
- 4, 1, −2, −5, … Yes
  -3, -3, -3



$5, 3, 1, -1, -3, \ldots$

Yes

General term of an arithmetic seq. is

$$a_n = a_1 + (n-1)d$$

first term

comm. dif

Ex: Find the first five terms of the seq. where $a_1 = 6$ and $a_n = a_{n-1} - 2$. Find the general term.

$a_1 = 6$, $a_2 = 6 - 2 = 4$, $a_3 = 2$, $a_4 = 0$, $a_5 = -2$

com. diff

$$\boxed{a_n = 6 + (n-1)(-2)}$$

$$\boxed{a_n = 6 - 2(n-1)}$$

Ex: Find a_8 of an arith. seq. whose first term is 4 and the common difference is -7.

$$a_n = 4 + (n-1)(-7)$$

$$a_8 = 4 + (8-1)(-7) = 4 + 7\cdot(-7) = 4 - 49$$
$$= \boxed{-45}$$

Ex: Find the general term of the arith. seq:

$$-2, 8, 18, 28, \ldots$$

$+10$ $+10$

$$a_n = -2 + (n-1)\cdot 10$$

$$\boxed{a_n = -2 + 10(n-1)}$$

$$= -2 + 10n - 10 = \boxed{10n - 12}$$

Ex: Find the sum of the first 99 terms of the arith. seq.

$$1, 2, 3, 4, \ldots$$

$$a_n = 1 + (n-1)\cdot 1 = 1 + n - 1 = \boxed{n}$$

99

Find $\sum_{i=1} i = 1+2+3+4+\dots+96+97+98+99$

$$
\begin{array}{l}
= 1 + 2 + 3 + 4 + \dots + 49 \quad +50 = 100 \cdot 0.5 \\
 99 + 98 + 97 + 96 + \dots + 51 \\
\hline
\underbrace{100 + 100 + 100 + 100 + \dots + 100} \quad +50
\end{array}
$$

$49 \cdot 100 + 50 = 4900 + 50$

$= \boxed{4950}$

$49 \cdot 100 + 0.5 \cdot 100$

$= 49.5 \cdot 100$

sum of the first and last term

number of terms divided by 2.

Sum of an arith. seq.

If $a_n = a_1 + (n+1)d$, then

$$\boxed{\sum_{i=1}^{N} a_i = \frac{N}{2}(a_1 + a_N)}$$

Ex: Find the sum of the first 100 terms of the seq: $1, 3, 5, 7, \dots$

$a_1 = 1$, $a_{100} = ?$

$\hookrightarrow a_n = 1 + (n-1)\cdot 2$

$a_{100} = 1 + 99\cdot 2 = 199$

The sum $S_{100} = \sum_{i=1}^{100} a_n = \frac{100}{2}(1+199)$

$= 50\cdot 200 = \boxed{10000}$

Ex: Find $\sum_{i=1}^{25} (5i - 9)$

is this an arith. seq?

$i=1 \Rightarrow 5\cdot 1 - 9 = -4$

$i=2 \Rightarrow 5\cdot 2 - 9 = 1$ +5

$i=3 \Rightarrow 5\cdot 3 - 9 = 6$ +5

Yes.

Any linear function will correspond to an arith. seq.

last term:

$i=25 \Rightarrow 5\cdot 25 - 9$

$= 116$

$$\sum_{i=1}^{25} (5i-9) = S_{25} = \frac{25}{2}(-4+116)$$

$$= \frac{25}{\cancel{2}}(\cancel{112}^{\,56}) = 25\cdot 56 = \boxed{1400}$$

Ex: Find the general term of the arith. seq with $a_2 = 3$ and $a_{14} = 51$

$$a_2 = a_1 + (2-1)d = \boxed{a_1 + d = 3}$$

$$a_{14} = a_1 + (14-1)d = \boxed{a_1 + 13d = 51}$$

$$
\begin{array}{r}
a_1 + 13d = 51 \\
-(a_1 + d = 3) \\
\hline
12d = 48
\end{array}
$$

$$d = 48/12 = \boxed{4} \rightarrow a_1 + 4 = 3$$

$$\boxed{a_1 = -1}$$

$$\boxed{a_n = -1 + (n-1)4}$$

How many terms will add to 2300?

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$2300 = \frac{n}{2}(-1 + (-1 + (n-1)4))$$

$$2300 = \frac{n}{2}(-1 - 1 + 4n - 4)$$

$$2300 = \frac{n}{2}(4n - 6)$$

$$2300 = 2n^2 - 3n$$

$$0 = 2n^2 - 3n - 2300$$

$$\vdots$$

$$n = \boxed{40}, -\frac{115}{3}$$