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Wednesday, November 29, 2017 10:30 AM

Section 11.3

Def: A geometric sequence is a sequence in which each term after the first is obtained by multiplying the preceding term by a fixed nonzero constant.

$\uparrow$ common ratio

seq	arith / geom. / neither
$1, 3, 5, 7, 9, 11, \ldots$ ($+2$; $d=2$)	arith
$1, 3, 9, 27, \ldots$ ($\times 3$, $\times 3$, $\times 3$)	geom, com. ratio is 3, $r=3$
$\frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \ldots$ ($\times 2$, $\times 2$)	geom, $r=2$
$-10, 2, -\frac{2}{5}, \frac{2}{25}, \ldots$	geom, $r=-\frac{1}{5}$

$\frac{2}{-10} = -\frac{1}{5}$

$$\frac{\frac{2}{25}}{-\frac{2}{5}} = \frac{\cancel{2}}{\underset{5}{\cancel{25}}} \cdot \frac{\cancel{5}}{-2} = \frac{-1}{5} \checkmark$$

$-\frac{2}{5}$ $\frac{28}{5}$ -4 ✓

$\overset{+6}{\frown}$ $\overset{-4}{\frown}$ $\longrightarrow$ not arith.

$-4, 2, -2, 4, 2, \ldots$

$\frac{2}{-4} = -\frac{1}{2}$

$\frac{-2}{2} = -1$ } not geom.

Geom. seq. - recursive formula

a_1 - first term, r - comm. ratio

$a_n = r \cdot a_{n-1}$, for $n \geq 2$.

General formula:

a_1 - first term, r - comm. ratio

$$a_n = a_1 \cdot r^{n-1}$$

Ex: Find the first 4 terms of the geom. seq. $a_1 = 6$, $r = \frac{1}{3}$

$6, \; 6 \cdot \frac{1}{3} = 2, \; 2 \cdot \frac{1}{3} = \frac{2}{3}, \; \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}, \ldots$

Ex: Find the eight term of the geom. seq.

$a = -4$. $r = -2$

Ex: Find the eighth term of the geom. seq.

$a_1 = -4$, $r = -2$.

$$a_8 = a_1 \cdot r^{8-1} = (-4) \cdot (-2)^7 = -4 \cdot (-128) = \boxed{512}$$

Ex: Find the general term formula for:

$$\overset{\times 2}{3, 6}, \underset{\times 2}{12}, 24, 48, \ldots$$

a_1 $\quad r = 2$

$$\boxed{a_n = 3 \cdot 2^{n-1}}$$

Sum of a geom. seq.

$$S_n = \sum_{i=1}^{n} a_i$$, where a_n is a geom. seq.

$a_n = a_1 r^{n-1}$

$$S_n = a_1 + a_1 \cdot r + a_1 \cdot r^2 + a_1 \cdot r^3 + \ldots + a_1 r^{n-1}$$

$$\cancel{S_{n+1} = a_1 + a_1 r + a_1 r^2 + \ldots + a_1 r^{n-1} + a_1 \cdot r^n}$$

multiply both sides by r.

$$S_n \cdot r = a_1 \cdot r + a_1 \cdot r^2 + a_1 r^3 + \ldots + a_1 r^{n-1} + a_1 r^n$$

$$S_n \cdot r = \quad a_1 \cdot r + a_1 \cdot r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} + a_1 r^n$$

$$S_n - S_n r = a_1 + \cancel{a_1 r} + \cancel{a_1 r^2} + \cancel{a_1 r^3} + \cdots + \cancel{a_1 r^{n-1}}$$
$$- \cancel{a r} - \cancel{a_1 r^2} - \cancel{a_1 r^3} - \cdots - \cancel{a_1 r^{n-1}} - a_1 r^n$$

$$S_n - S_n r = a_1 - a_1 r^n$$
$$\frac{S_n(1-r)}{1-r} = \frac{a_1(1-r^n)}{1-r}$$

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}}$$

$r \neq 1$

What if $r = 1$?

$$a_n = a_1 \cdot 1^{n-1} = a_1$$

seq: $a_1, a_1, a_1, a_1, \ldots$

Find the sum of the first 18 terms of:

$2, -8, 32, -128, \ldots$

$\times -4$ $\quad$ $\times -4$

$a_1 = 2, \quad r = -4$

$$S_{18} = \frac{2(1-(-4)^{18})}{1-(-4)} = \boxed{\frac{2(1-4^{18})}{5}}$$

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10 $\quad$ $12(1-2^{10})$

Find : $\sum_{i=1}^{10} 6 \cdot 2^i$ $= S_{10} = \dfrac{12(1-2^{10})}{1-2}$

$6 \cdot 2^1 = 12$ — a_1

$6 \cdot 2^2 = 24$ $r=2$

$6 \cdot 2^3 = 48$

$= \dfrac{12(1-2^{10})}{-1}$

$= \boxed{12(2^{10}-1)}$

Find the general term of the geom. seq:

$3, a_2, a_3, -24, \ldots$

$a_4 = a_1 \cdot r^{4-1}$

$\dfrac{-24}{3} = \dfrac{3 \cdot r^3}{3}$

$\sqrt[3]{-8} = \sqrt[3]{r^3}$

$\boxed{-2 = r} \longrightarrow \boxed{a_n = 3 \cdot (-2)^{n-1}}$

$a_1 = 1$, $r = 2$

$S_{64} = \dfrac{1(1-2^{64})}{1-2} = \dfrac{1-2^{64}}{-1}$

$= \boxed{2^{64}-1}$

Formulas

	Arith	Geom.
recursive	$a_n = a_{n-1} + d$	$a_n = r\,a_{n-1}$
general	$a_n = a_1 + d(n-1)$	$a_n = a_1 \cdot r^{n-1}$
Sum of the first n terms	$S_n = \frac{n}{2}(a_1 + a_n)$	$S_n = \frac{a_1(1-r^n)}{1-r}$