Review • on Saturday, MMC CP 145, 10am-noon
• online office hour on Sunday at 6:30 PM.

Section 11.5

Def: For nonnegative integers r, n, where $n \geq r$, the expression $\binom{n}{r}$, (read "n above r") ("n choose r") is called the binomial coefficient and is defined by

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}.$$

Evaluate:

$$\binom{6}{2} = \frac{6!}{(6-2)!\cdot 2!} = \frac{6\cdot 5\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}\cdot 2\cdot 1} = \frac{6\cdot 5}{2} = \boxed{15}$$

$$\binom{3}{0} = \frac{3!}{(3-0)!\,0!} = \frac{\cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{3}\cdot \cancel{2}\cdot \cancel{1}\cdot 1} = \boxed{1}$$

3 4

$$\binom{9}{3} = \frac{9!}{(9-3)!\,3!} = \frac{9\cdot8\cdot7\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}\cdot 3\cdot2\cdot1} = \frac{\overset{3}{\cancel{9}}\cdot\overset{4}{8}\cdot7}{\cancel{3}\cdot\cancel{2}}$$

$$= 12\cdot 7 = \boxed{84}$$

$$\binom{4}{4} = \frac{4!}{(4-4)!\cdot 4!} = \frac{\cancel{4!}}{0!\cdot\cancel{4!}} = \frac{1}{0!} = \boxed{1}$$

Thm: (The Binomial theorem)

For any positive n:

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots$$

$$\dots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$$

Foil: $(a+b)^2 = \binom{2}{0}a^2b^0 + \binom{2}{1}a^{2-1}b^1 + \binom{2}{2}a^{2-2}b^2$

$$= \frac{2!}{(2-0)!\,0!}a^2 + \frac{2!}{(2-1)!\,1!}a^1b^1 + \frac{2!}{(2-2)!\,2!}a^0b^2$$

$$= 1\cdot a^2 + \frac{2}{1\cdot1}ab + 1\cdot b^2 = \underline{a^2 + 2ab + b^2}$$

Find the first 4 terms of $(x+3)^4$.

in a calculator: ${}_4C_2$

Find the first 4 terms of $(x+3)^4$.

$$\underbrace{\quad}_{}\ \text{in a calculator: } {}_4C_2$$

$$= \binom{4}{0} x^{4-0} \cdot 3^0 + \binom{4}{1} x^{4-1} 3^1 + \binom{4}{2} x^{4-2} \cdot 3^2 + \binom{4}{3} x^{4-3} \cdot 3^3 + \ldots$$

$$= \underline{1} \cdot x^4 \cdot 1 + \frac{4!}{3! \cdot 1!} \cdot x^3 \cdot 3 + \frac{4!}{2! \cdot 2!} x^2 \cdot 3^2 + \frac{4!}{1! \cdot 3!} x^1 \cdot 3^3 + \ldots$$

$$= x^4 + \frac{4 \cdot \cancel{3!}}{\cancel{3!}} x^3 \cdot 3 + \frac{24}{2 \cdot 2} x^2 \cdot 9 + 4 \cdot x \cdot 27 + \ldots$$

$$= x^4 + 4 \cdot 3x^3 + 6 \cdot 9x^2 + 4 \cdot 27x + \ldots$$

$$= \boxed{x^4 + 12x^3 + 54x^2 + 108x} + \ldots$$

Find the first 3 terms of $(2x-y)^5$

$$(2x-y)^5 = \binom{5}{0} (2x)^{5-0} (-y)^0 + \binom{5}{1} (2x)^4 (-y)^1 + \binom{5}{2} (2x)^3 (-y)^2 + \ldots$$

$$= 1 \cdot 2^5 x^5 \cdot 1 + 5 \cdot 2^4 x^4 \cdot (-y) + \frac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1} \cdot \cancel{2} \cdot 1} 2^3 x^3 \cdot y^2 + \ldots$$

$$= 32x^5 - 5 \cdot 16 x^4 y + 5 \cdot 2 \cdot 8 \cdot x^3 \cdot y^2 + \ldots$$

$$= \boxed{32x^5 - 80x^4 y + 80x^3 y^2} + \ldots$$

Bin. thm:

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{n} a^0 b^n$$

$$= \boxed{\sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^{i}}$$

Note: The $(r+1)^{th}$ term of the expansion $(a+b)^n$ is $\binom{n}{r} a^{n-r} b^{r}$.

Ex: Find the fourth term of the expansion $(3x+2y)^7$

$\searrow i=3$

$$\binom{7}{3} (3x)^{7-3} (2y)^{3} = \frac{7!}{4!\,3!}\, 3^4 \cdot x^4 \cdot 2^3 \cdot y^3$$

$$= \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} \cdot 3^4 \cdot 2^3 \cdot x^4 \cdot y^3$$

$$= \boxed{7 \cdot 5 \cdot 81 \cdot 8 \cdot x^4 y^3}$$

Ex: Find the term containing x^6 in the expansion of $(2x-y)^9 = \sum_{i=0}^{n} \binom{n}{i} 2^{n-i} x^{n-i} \cdot (-y)^i$

$$6 = 9 - i \rightarrow \underline{i=3}$$

$$\binom{9}{3} (2x)^6 (-y)^3 = \frac{\overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6!}}{\cancel{6!} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} \cdot 2^6 x^6 (-y^3)$$

$$= -3 \cdot 4 \cdot 7 \cdot 64\, x^6 y^3$$

$$= -3 \cdot 4 \cdot 7 \cdot 64\, x^6 y^3$$

$$= \boxed{-84 \cdot 64\, x^6 y^3}$$