12/03

Sunday, December 3, 2017 6:32 PM

Review

3 a) $\frac{3}{2}, \frac{3}{8}, \frac{3}{32}, \ldots$ geom. seq.

$r$ $r$

$r = \frac{\frac{3}{8}}{\frac{3}{2}} = \frac{\cancel{3}}{\cancel{8}_4} \cdot \frac{\cancel{2}}{\cancel{3}} = \boxed{\frac{1}{4}} \rightarrow a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = \frac{3}{2} \cdot \left(\frac{1}{4}\right)^{n-1}}$

To find $a_4$, • $a_4 = \frac{3}{2} \cdot \left(\frac{1}{4}\right)^{4-1} = \frac{3}{2} \cdot \frac{1}{4^3} = \frac{3}{2 \cdot 64} = \boxed{\frac{3}{128}}$

• using the recursive formula:

$a_n = r \cdot a_{n-1}$

$a_4 = r \cdot a_3 = \frac{1}{4} \cdot \frac{3}{32} = \frac{3}{4 \cdot 32} = \boxed{\frac{3}{128}}$

$a_5 = r \cdot a_4 = \frac{1}{4} \cdot \frac{3}{128} = \frac{3}{4 \cdot 128} = \boxed{\frac{3}{512}}$

b) $-5, -10, -20, -40, \ldots$

$\times 2$ $\times 2$

$\boxed{r = 2}$

$r = \frac{-10}{-5} = 2$

$r = \frac{-20}{-10} = 2$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = -5 \cdot 2^{n-1}}$

$\boxed{a_n = r \cdot a_{n-1}}$

$a_5 = 2 \cdot a_4$

$= 2 \cdot (-40) = \boxed{-80}$

$a_5 = 2 \cdot a_4$

$= 2 \cdot (-40) = \boxed{-80}$

$a_6 = 2 \cdot (-80) = \boxed{-160}$

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(6) Find the coefficient of $x^5$ in $(2x-3)^9$.

$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$

$$(2x-3)^9 = \sum_{r=0}^{9} \binom{9}{r} (2x)^{9-r} \cdot (-3)^r$$

$x^5$ $\quad 9-r=5$

$r = 9-5 = 4$

$$\binom{9}{4} (2x)^5 \cdot (-3)^4 = \frac{9!}{(9-4)!\,4!} \cdot 2^5 x^5 3^4$$

$$= \frac{9 \cdot \overset{2}{\cancel{8}} \cdot 7 \cdot \cancel{6} \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \cancel{4 \cdot 3 \cdot 2} \cdot 1} \cdot 2^5 3^4 \cdot x^5$$

$$= \boxed{9 \cdot 2 \cdot 7 \cdot 2^5 \cdot 3^4 \cdot x^5}$$

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(8)

express in sigma notation:

$$\frac{\overset{= r^0}{1}}{1y} + \frac{r^1}{2y} + \frac{r^2}{3y} + \dots + \frac{r^{n-1}}{ny} = \boxed{\sum_{k=1}^{n} \frac{r^{k-1}}{k \cdot y}}$$

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(10) a) $\sum_{i=3}^{6} (3i-5) = 3 \cdot \underline{3} - 5 + 3 \cdot \underline{4} - 5 + 3 \cdot \underline{5} - 5 + 3 \cdot \underline{6} - 5$

(11) a) $\sum_{i=3}^{6}(3i-5) = 3\cdot\underline{3}-5 + 3\cdot\underline{4}-5 + 3\cdot\underline{5}-5 + 3\cdot\underline{6}-5$

$= \ldots$

b) $\sum_{i=0}^{4}\frac{(i+2)!}{i!} = \frac{(0+2)!}{0!} + \frac{(1+2)!}{1!} + \frac{(2+2)!}{2!} + \frac{(3+2)!}{3!} + \frac{(4+2)!}{4!}$

$= \frac{2!}{0!} + \frac{3!}{1!} + \frac{4!}{2!} + \ldots$

$= \frac{2\cdot1}{1} + \frac{3\cdot2\cdot1}{1} + \frac{4\cdot3\cdot\cancel{2\cdot1}}{\cancel{2\cdot1}} + \ldots$

$= 2 + 6 + 12 + \ldots$

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(12) Arith. seq. $a_{10} = -21$, $a_{16} = -39$

$a_n = a_1 + d(n-1)$

$-21 = a_1 + d(10-1)$

$-39 = a_1 + d(16-1)$

$-21 = a_1 + 9d$

$-(-39 = a_1 + 15d)$

$-21+39 = 9d - 15d$

$18 = -6d$

$d = -\frac{18}{6} = \boxed{-3}$

$-21 = a_1 + 9\cdot(-3)$

$-21 = a_1 - 27$

$\boxed{a_n = 6 - 3(n-1)}$

$\boxed{a_n = 6 - 3(n-1)}$

$-21 = a_1 - 27$

$\boxed{a_1 = 6}$

$a_n = a_{n-1} + d$

$\boxed{a_n = a_{n-1} - 3}$

seq: $6, 3, 0, -3, -6, -9, \ldots$

(14) Find the coeff. of $x^8$ in $(x^2-3)^7$

$$(x^2-3)^7 = \sum_{r=0}^{7} \binom{7}{r} \underbrace{(x^2)^{7-r}}_{= x^8} \cdot (-3)^r$$

$$(x^2)^{7-r} = x^8$$

$$x^{2(7-r)} = x^8$$

$$x^{14-2r} = x^8$$

$$\downarrow$$

$$14 - 2r = 8$$

$$2r = 6$$

$$\boxed{r = 3}$$

$$\underbrace{\binom{7}{3}}_{= \frac{7!}{(7-3)!\,3!}} \cdot (x^2)^{7-3} \cdot \underbrace{(-3)^3}_{= -3^3} = \frac{7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{3} \cdot \cancel{2} \cdot 1} \quad x^8 \cdot (-3^3)$$

$$= 7 \cdot 5 \cdot x^8 \cdot (-27) = \boxed{-27 \cdot 7 \cdot 5\, x^8}$$

(15) $\sum_{k=0}^{n-1}(3k+1) = \boxed{3\cdot 0+1+3\cdot 1+1+3\cdot 2+1+\cdots+3(n-1)+1}$

(17) b) Expess in sigma notation:

$$\frac{a+1}{1}+\frac{a+2}{2}+\frac{a+3}{3}+\cdots+\frac{a+6}{6} = \sum_{k=1}^{6}\frac{a+k}{k}$$

$$= \sum_{i=1}^{6}\frac{a+i}{i}$$

(21) Evaluate: $\binom{10}{5} = \frac{10!}{(10-5)!\,5!} = \frac{\cancel{10}\cdot \overset{3}{\cancel{9}}\cdot \overset{2}{\cancel{8}}\cdot 7\cdot 6\cdot \cancel{5!}}{\cancel{5!}\cdot \cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 3\cdot 2\cdot 7\cdot 6$

$= 6\cdot 6\cdot 7 = 36\cdot 7 = \boxed{252}$

$$\begin{array}{r} 36 \\ \cdot\, 7 \\ \hline 252 \end{array}$$

(22) Find $a_1, a_2, a_3, a_4$ of $a_1 = 2,\ a_2 = 5$

$a_n = a_{n-2} - 3a_{n-1}$

$\underline{n=3}$

$a_3 = a_1 - 3\cdot a_2 = 2 - 3\cdot 5 = 2 - 15 = \boxed{-13}$

$\underline{n=4}$

$a_4 = a_2 - 3\cdot a_3 = 5 - 3\cdot(-13) = 5 + 39 = \boxed{44}$

(32) Find $a_2, a_3$ in a geom. seq. $\underset{= a_1}{2}, a_2, a_3, \underset{= a_4}{\frac{3}{32}}$

$a_n = a_1 r^{n-1}$

$2 = a_1 \to 2 = a_1 \cdot r^{1-1} \to \underline{2 = a_1}$

$\frac{3}{32} = a_4 \to \frac{3}{32} = a_1 \cdot r^{4-1} \to \underline{\frac{3}{32} = a_1 \cdot r^3}$

$$\frac{\frac{3}{32}}{2} = \frac{2 \cdot r^3}{2}$$

$$\sqrt[3]{\frac{3}{64}} = \sqrt[3]{r^3}$$

$$r = \frac{\sqrt[3]{3}}{\sqrt[3]{64}} = \boxed{\frac{\sqrt[3]{3}}{4}}$$

$$a_2 = a_1 \cdot r = 2 \cdot \frac{\sqrt[3]{3}}{4} = \boxed{\frac{\sqrt[3]{3}}{2}}$$

$$a_3 = a_2 r = \frac{\sqrt[3]{3}}{2} \cdot \frac{\sqrt[3]{3}}{4} = \frac{(\sqrt[3]{3})^2}{8} = \boxed{\frac{\sqrt[3]{9}}{8}}$$

Find $\sum_{k=5}^{33} (3k-1) = 3\cdot 5-1 + 3\cdot 6-1 + 3\cdot 7-1 + \dots + 3\cdot 33-1$

$n = 33-5+1$

$= 14 + 17 + 20 + \dots + 98$

$n = 33 - 5 + 1$

or

$= \underbrace{14 + 17 + 20 + \ldots + 98}_{a_1 = 14 \quad d = 3}$

$+3 \quad +3$

$98 = 14 + 3(n-1)$

$84 = 3n - 3$

$87 = 3n$

$n = \frac{87}{3} = 29$

$a_n = 14 + 3(n-1)$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_? = \frac{?}{2}(14 + 98)$

$S_{29} = \frac{29}{2}(14 + 98) = \frac{29}{\cancel{2}} \cdot \cancel{112}^{56}$

$= \boxed{29 \cdot 56} = \boxed{1624}$