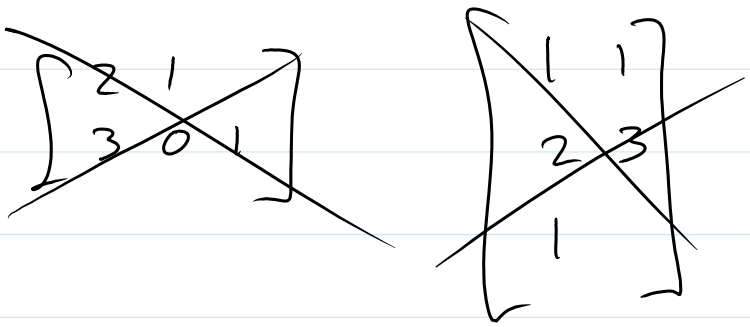
Section 9.5

A matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$

2 rows
2 columns

2 rows
3 columns



Def: A determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is denoted as

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - b \cdot c$$

$-$ $+$

Find:

- $\begin{vmatrix} 5 & 6 \\ 7 & 3 \end{vmatrix} = 5 \cdot 3 - 6 \cdot 7 = 15 - 42 = \boxed{-27}$

$$\cdot \begin{vmatrix} 2 & 4 \\ -3 & -5 \end{vmatrix} = 2\cdot(-5) - 4(-3) = -10 + 12 = \boxed{2}$$

Solving a system of two equations using the Cramer's rule

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases} \qquad \begin{cases} 2x - 3y = 0 \\ x + 2y = 5 \end{cases}$$

let $D = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$, $D_x = \begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}$, $D_y = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}$

$D = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$, $D_x = \begin{bmatrix} 0 & -3 \\ 5 & 2 \end{bmatrix}$, $D_y = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$

the solution (if there is one) is

$$x = \frac{|D_x|}{|D|} \quad , \quad y = \frac{|D_y|}{|D|}$$

$|D| = 4 + 3 = 7$, $|D_x| = 0 + 15 = 15$, $|D_y| = 10 - 0 = 10$

$$\boxed{x = \frac{15}{7} \quad , y = \frac{10}{7}}$$

Ex: Solve using the Cramer's rule

$$\begin{cases} 5x - 4y = 2 \\ 6x - 5y = 1 \end{cases}$$

$$D = \begin{vmatrix} 5 & -4 \\ 6 & -5 \end{vmatrix} = -25 + 24 = -1$$

$$x = \frac{D_x}{D} = \frac{-6}{-1} = 6$$

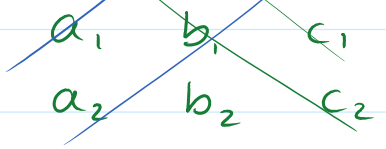
$$D_x = \begin{vmatrix} 2 & -4 \\ 1 & -5 \end{vmatrix} = -10 + 4 = -6$$

$$y = \frac{D_y}{D} = \frac{-7}{-1} = 7$$

$$D_y = \begin{vmatrix} 5 & 2 \\ 6 & 1 \end{vmatrix} = 5 - 12 = -7$$

Determinant of a 3x3 matrix

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \cdot b_2 \cdot c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - a_3 b_2 c_1 - a_1 b_3 c_2 - a_2 b_1 c_3$$



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