12/08

Friday, December 8, 2017 11:05 AM

Section 9.5

Ex: Use the Cramer's rule to solve:

$$\begin{cases} x - y = 1 \\ 2x + 2y = 3 \end{cases}$$

$$D = \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = 1 \cdot 2 - (-1)2 = 2 + 2 = 4$$

$$D_x = \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 1 \cdot 2 - (-1)3 = 2 + 3 = 5$$

$$D_y = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1 \cdot 3 - 1 \cdot 2 = 3 - 2 = 1$$

$$x = \frac{D_x}{D} = \boxed{\frac{5}{4}} \qquad y = \frac{D_y}{D} = \boxed{\frac{1}{4}}$$

$$\boxed{\left(\frac{5}{4}, \frac{1}{4} \right)}$$

Ex: Evaluate:

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix} = 1\cdot 2\cdot 1 + 0\cdot 1\cdot 2 + 3\cdot 0\cdot 1 - 2\cdot 2\cdot 3 - 1\cdot 1\cdot 1 - 1\cdot 0\cdot 0$$

$$= 2 + 0 + 0 - 12 - 1 - 0 = \boxed{-11}$$

$$\begin{matrix} 1 & 0 & 2 \\ 0 & 2 & 1 \end{matrix}$$

Def: If A is $n\times n$ matrix, then

$$|A| = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} M_{ij},$$

where a_{ij} is the element in i^{th} row and j^{th} column and M_{ij} is a minor of A, i.e.,

$$M_{ij} = \underbrace{\begin{vmatrix} a_{11} & a_{12} & a_{12} \cdots \\ a_{21} & a_{22} & a_{23} \\ \vdots & \vdots & \vdots \end{vmatrix}}_{A} \begin{matrix} \\ i^{\text{th}} \\ \\ \end{matrix}$$

j^{th}

M_{ij} = "the matrix A without the j^{th} column and i^{th} row".

Use the formula above to find

1st column

$$|A| = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix}$$

1) Pick a row/column with the most # of zeros

$$|A| = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix}$$

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$$|A| = (-1)^{1+1}\cdot 1\cdot \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + (-1)^{2+1}\cdot 0\cdot \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} + (-1)^{3+1}\cdot 3\cdot \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= (-1)^{2}\cdot 1\cdot(2-1) + 0 + (-1)^{4}\cdot 3\cdot(0-4)$$

$$= 1 + 0 + (-12) = \boxed{-11}$$

2col

$$|A| = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix}$$

$$= (-1)^{1+2}\cdot 0\cdot \begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} + (-1)^{2+2}\cdot 2\cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + (-1)^{3+2}\cdot 1\cdot \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

Solve:

$$\begin{cases} x + 2y - z = -4 \\ x + 4y - 2z = -6 \\ 2x + 3y + z = 3 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix}, \quad D_x = \begin{vmatrix} -4 & 2 & -1 \\ -6 & 4 & -2 \\ 3 & 3 & 1 \end{vmatrix}, \quad D_y = \begin{vmatrix} 1 & -4 & -1 \\ 1 & -6 & -2 \\ 2 & 3 & 1 \end{vmatrix}, \quad D_z = \begin{vmatrix} 1 & 2 & -4 \\ 1 & 4 & -6 \\ 2 & 3 & 3 \end{vmatrix}$$

$$\begin{array}{cccc} & 1 & 2 & -1 \\ & 1 & 4 & -2 \\ & 2 & 3 & 1 \\ - & 1 & 2 & -1 \quad + \\ & 1 & 4 & -2 \end{array}$$

$$D = 4 + (-3) + (-8) - (-8) - (-6) - 2$$
$$= 4 - 3 - \cancel{8} + \cancel{8} + 6 - 2$$
$$= \boxed{5}$$

$$D_x = \begin{vmatrix} -4 & 2 & -1 \\ -6 & 4 & -2 \\ 3 & 3 & 1 \end{vmatrix} = -16 + 18 - 12 - (-12) - (24) - (-12)$$
$$\begin{array}{ccc} -4 & 2 & -1 \\ -6 & 4 & -2 \end{array}$$
$$= -16 + 18 - 12 + \cancel{12} - \cancel{24} + \cancel{12}$$
$$= \boxed{-10}$$

$$x = \frac{D_x}{D} = \frac{-10}{5} = \boxed{-2}, \qquad y = \frac{D_y}{D}, \qquad z = \frac{D_z}{D}$$

$$D_y = \begin{vmatrix} 1 & -4 & -1 \\ 1 & -6 & -2 \\ 2 & 3 & 1 \end{vmatrix} = -6 + (-3) + 16 - 12 - (-6) - (-4)$$
$$\begin{array}{cccc} - & 1 & -4 & -1 \quad + \\ & 1 & -6 & -2 \end{array}$$
$$= -\cancel{6} - 3 + 16 - 12 + \cancel{6} + 4$$
$$= 5$$

$$D_z = \begin{vmatrix} 1 & 2 & -4 \\ 1 & 4 & -6 \\ 2 & 3 & 3 \end{vmatrix} = 12 + (-12) + (-24) - (-32) - (-18) - 6$$

$$\begin{matrix} 1 & 2 & -4 \\ 1 & 4 & -6 \end{matrix}$$

$$= \cancel{12} - \cancel{12} - 24 + 32 + 18 - 6$$

$$= 20$$

$$z = \frac{D_z}{D} = \frac{20}{5} = \boxed{4}$$

$$y = \frac{D_y}{D} = \frac{5}{5} = \boxed{1}$$