

Section 2.6

Ex: $f(x) = 2x - 1$, $g(x) = x^2 + x - 2$. Find:

Domains: $(-\infty, \infty)$

$$\begin{aligned} \text{a) } (f+g)(x) &= (2x-1) + (x^2+x-2) \\ &= 2x-1 + x^2+x-2 = \boxed{x^2+3x-3} \end{aligned}$$

$$\begin{aligned} \text{b) } (f-g)(x) &= (2x-1) - (x^2+x-2) \\ &= 2x-1 - x^2-x+2 = \boxed{-x^2+x+1} \end{aligned}$$

$$\begin{aligned} \text{c) } (f \cdot g)(x) &= (2x-1)(x^2+x-2) \\ &= 2x^3 + 2x^2 - 4x - x^2 - x + 2 \\ &= \boxed{2x^3 + x^2 - 5x + 2} \end{aligned}$$

The domain for each function above is $(-\infty, \infty)$

$$\text{d) } \left(\frac{f}{g}\right)(x) = \frac{2x-1}{x^2+x-2} = \frac{2x-1}{(x+2)(x-1)}$$

$$\begin{aligned} x+2 &= 0 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} x-1 &= 0 \\ x &= 1 \end{aligned}$$

↳ Domain:

$$(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$$

Ex: $f(x) = \sqrt{x+3}$

Dom: $[-3, \infty)$

$g(x) = \sqrt{x-2}$

Dom: $[2, \infty)$

$$\text{Find } (f+g)(x) = \sqrt{x+3} + \sqrt{x-2} = \sqrt{x+3+x-2}$$

$$\text{Dom: } [2, \infty) = [-3, \infty) \cap [2, \infty)$$

↑
All numbers that are in both intervals

$$f(x) = \sqrt{x+3}$$

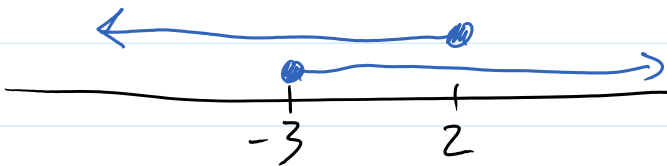
$$\text{Dom: } [-3, \infty)$$

$$g(x) = \sqrt{2-x}$$

$$\text{Dom: } (-\infty, 2]$$

$$\text{Find } (f+g)(x) = \sqrt{x+3} + \sqrt{2-x}$$

$$\text{Dom: } [-3, \infty) \cap (-\infty, 2] = [-3, 2]$$



Function composition

Ex:

$$f(x) = \frac{2}{x-1}, \quad g(x) = \frac{3}{x}$$

Find:

$$\begin{aligned} \text{a) } (f \circ g)(x) &= f(g(x)) = f\left(\frac{3}{x}\right) = \frac{2}{\frac{3}{x} - 1 \cdot \frac{x}{x}} \\ &= \frac{2}{\frac{3-x}{x}} = \frac{2}{\frac{3-x}{x}} \end{aligned}$$

$$= \frac{3}{x} - \frac{x}{x} = \frac{3-x}{x}$$

$$= \frac{\frac{2}{1}}{\frac{3-x}{x}} = \frac{2}{1} \cdot \frac{x}{3-x} = \boxed{\frac{2x}{3-x}}$$

Dom: $(-\infty, 3) \cup (3, \infty)$

b) $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{2}{x-1}\right) = \frac{\overset{+3}{2}}{\cancel{x-1}} \cdot \frac{x-1}{\cancel{x-1}} = \boxed{\frac{3(x-1)}{2}}$$

one-to-one

