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Wednesday, September 27, 2017 10:57 AM

Section 3.3 - Polynomial division

Review: Divide 21345 by 12

$$
\begin{array}{r}
1778 \\
12\,\overline{)\,21345} \\
-12 \\
\hline
93 \\
-84 \\
\hline
94 \\
-84 \\
\hline
105 \\
-96 \\
\hline
9
\end{array}
\qquad \rightleftharpoons \qquad \frac{21345}{12} = 1778 + \frac{9}{12}
$$

remainder

$$
\begin{array}{r}
x + 7 \\
x+3\,\overline{)\,x^2 + 10x + 21} \\
-(x^2 + 3x) \\
\hline
7x + 21 \\
-(7x + 21) \\
\hline
0x + 0
\end{array}
\qquad \rightleftharpoons \qquad \frac{x^2+10x+21}{x+3} = x + 7
$$

Ex: Divide $4 - 5x - x^2 + 6x^3$ by $3x - 2$

$\frac{6x^3}{3x}$ $\quad \frac{3x^2}{3x}$ $\quad \frac{-3x}{3x}$

$$2x^2 + x - 1$$

$$
\begin{array}{r}
2x^2 + x - 1 \\
3x-2 \overline{\smash{\big)}\, 6x^3 - x^2 - 5x + 4} \\
\underline{-6x^3 + 4x^2 \quad\qquad} \\
3x^2 - 5x \quad \\
\underline{-3x^2 + 2x \quad} \\
-3x + 4 \\
\underline{+3x - 2} \\
2
\end{array}
$$

remainder

$$\frac{6x^3 - x^2 - 5x + 4}{3x-2} = 2x^2 + x - 1 + \frac{2}{3x-2}$$

$$6x^3 - x^2 - 5x + 4 = (3x-2)(2x^2+x-1) + 2$$

Ex:

Divide: $6x^4 + 5x^3 + 3x - 5$ by $3x^2 - 2x$

$$
\begin{array}{r}
2x^2 + 3x + 2 \\
3x^2-2x \overline{\smash{\big)}\, 6x^4 + 5x^3 + 0x^2 + 3x - 5} \\
\underline{-6x^4 + 4x^3 \qquad\qquad\qquad} \\
9x^3 + 0x^2 + 3x - 5 \\
\underline{-9x^3 + 6x^2 \qquad\qquad} \\
6x^2 + 3x - 5 \\
\underline{-6x^2 + 4x \quad\;\;} \\
7x - 5
\end{array}
$$

$6x^4 + 5x^3 + 3x - 5 \quad 2x^2 + 3x + 2 + \frac{7x-5}{\ldots}$

$$\frac{6x^4+5x^3+3x-5}{3x^2-2x} = 2x^2+3x+2 + \underbrace{\frac{7x-5}{3x^2-2x}}_{}$$

$$6x^4+5x^3+3x-5 = \underbrace{(3x^2-2x)}_{\text{divisor}}\underbrace{(2x^2+3x+2)}_{\text{Quotient}} + \overbrace{7x-5}^{\text{remainder}}$$

Note: Evaluating $6x^4+5x^3+3x-5$ at $x=0$, we replace x with 0 on the left side.

What is the function value at $x=\frac{2}{3}$?
Replace x with $2/3$ on the right side:

$$(3x^2-2x)(2x^2+3x+2)+7x-5$$
$$= x(3x-2)(2x^2+3x+2)+7x-5$$

$x=2/3$

$$= \underbrace{\frac{2}{3}\underbrace{\left(3\cdot\frac{2}{3}-2\right)}_{=0}\left(2\cdot\left(\frac{2}{3}\right)^2+3\left(\frac{2}{3}\right)+2\right)}_{=0} + 7\cdot\left(\frac{2}{3}\right)-5$$

$$+ 7\cdot\frac{2}{3}-5$$

$$= \frac{14}{3}-5\cdot\frac{3}{3} = \frac{14}{3}-\frac{15}{3} = \boxed{-1/3}$$

Synthetic division

$$\begin{array}{r} x^2+7x+16 \\ x-3\,\overline{)\,x^3+4x^2-5x+5} \\ -\,x^3+3x^2 \\ \hline 7x^2-5x \end{array}$$

	x^3	x^2	x	const
	1	4	-5	5
3	$\downarrow$	$3\cdot1$ 3	$3\cdot7$ 21	$3\cdot16$ 48
	1	$4+3$ 7	$21-5$ 16	$48+5$ 53

$-x+3x$ $\qquad$ 1 | 7 | 16 | 53

$$\begin{array}{r} 7x^2-5x \\ \underline{-7x^2+21x} \\ 16x+5 \\ \underline{-16x+48} \\ 53 \end{array}$$

Divide: $5x^3+6x+8$ by $x+2$

$5x^3+0x^2+6x+8$ $\qquad$ $x-(-2)$

	5	0	6	8
-2		-10	20	-52
	5	-10	26	-44

$\leftarrow 5x^2-10x+26$

remainder

Thm If the polynomial $f(x)$ is divided by $x-c$, then the remainder is $f(c)$.

$$f(x)=5x^3+6x+8=(x-(-2))(5x^2-10x+26)+(-44)$$

$$f(-2)=\boxed{-44}$$

Thm: let $f(x)$ be polynomial:

a) If $f(c)=0$, then $x-c$ is a factor of $f(x)$

b) If $x-c$ is a factor of $f(x)$, then $f(c)=0$.
