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Friday, September 29, 2017 10:59 AM

Ex: $\dfrac{2x^5-3x^4+4x^2-7}{2x-1}$

$$
\begin{array}{r}
x^4 - x^3 - \frac{1}{2}x^2 + \frac{7}{4}x + \frac{7}{8} \\
2x-1 \,\big)\, 2x^5 - 3x^4 + 0x^3 + 4x^2 + 0x - 7 \\
\underline{-2x^5 + x^4} \\
-2x^4 + 0x^3 + 4x^2 \\
\underline{+2x^4 \mp x^3} \\
-x^3 + 4x^2 + 0x - 7 \\
\underline{+x^3 \mp \frac{1}{2}x^2} \\
\frac{7}{2}x^2 + 0x - 7 \\
\underline{-\frac{7}{2}x^2 + \frac{7}{4}x} \\
\frac{7}{4}x - 7 \\
\underline{-\frac{7}{4}x + \frac{7}{8}} \\
\boxed{-\frac{49}{8}}
\end{array}
$$

remainder

$\dfrac{-x^3}{2x} = \dfrac{-x^2}{2}$

$\dfrac{\frac{7x^2}{2}}{\frac{2x}{1}} = \dfrac{7x^2}{2} \cdot \dfrac{1}{2x} = \dfrac{7x}{4}$

$\dfrac{7\cancel{x}}{4} \cdot \dfrac{1}{2\cancel{x}} = \dfrac{7}{8}$

Divide: $(6x^5-2x^3+4x^2-3x+1)$ by $(x-2)$

	x^5	x^4	x^3	x^2	x	const
	6	0	-2	4	-3	1
2	↓	12	24	44	96	186
	6	12	22	48	93	187

6	12	22	48	93	187
x^4	x^3	x^2	x	const	↑ remainder

Divide: $(x^3 - 2x^2 - 5x + 6)$ by $(3 - x)$

$$\frac{x^3 - 2x^2 - 5x + 6}{3 - x} = -\left(\frac{x^3 - 2x^2 - 5x + 6}{x - 3}\right)$$

	1	-2	-5	6
3		3	3	-6
	1	1	-2	0

$$-\left(\frac{x^3 - 2x^2 - 5x + 6}{x - 3}\right) = -\left(x^2 + x - 2\right) = \boxed{-x^2 - x + 2}$$

Section 3.4

Thm: The Rational Zero Thm

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ has integer coefficients and $\frac{p}{q}$ is a root of this polynomial (rational zero), then p is a factor of the constant term a_0, and q is a factor of a_n

p is a factor of the constant term a_0, and q is a factor of a_n.

Ex: list all posible rational zeros of

$$f(x) = -x^4 + 3x^2 + 4$$

$4: \pm 1, \pm 2, \pm 4$

$-1: \pm 1$

$$\frac{\pm 1, \pm 2, \pm 4}{\pm 1} = \boxed{\pm 1, \pm 2, \pm 4}$$

Ex: list all possible rational zeros of:

$$f(x) = 15x^3 + 14x^2 - 3x - 2$$

$-2: \pm 1, \pm 2$

$15: \pm 1, \pm 3, \pm 5, \pm 15$

$$\frac{\pm 1, \pm 2}{\pm 1, \pm 3, \pm 5, \pm 15} \Rightarrow \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{1}{15}, \pm \frac{2}{15}$$

Factor: $f(x) = x^3 + 2x^2 - 5x - 6$

$$\left.\begin{array}{l} -6: \pm 1, \pm 2, \pm 3, \pm 6 \\ 1: \pm 1 \end{array}\right\} \text{possible zeros: } \pm 1, \pm 2, \pm 3, \pm 6$$

$f(1) = 1 + 2 - 5 - 6 \neq 0$

$f(-1) = -1 + 2 + 5 - 6 = 0 \rightarrow (x - (-1))$ is a factor

	1	2	-5	-6
-1		-1	-1	6
	1	1	-6	0

$\underbrace{1 \quad 1 \quad -6}_{x^2+x-6}$

$$x^3+2x^2-5x-6=(x-(-1))\,(x^2+x-6)$$
$$=\boxed{(x+1)\,(x+3)\,(x-2)}$$

Ex: Find all zeros of $f(x)=x^3+7x^2+11x-3$

$-3: \pm 1, \pm 3$
$1: \pm 1$
$\Big\}\ \dfrac{\pm 1, \pm 3}{\pm 1} \to \pm 1, \pm 3$

	1	7	11	-3	
1		1	8	19	
	1	8	19	16	X

	1	7	11	-3	
-1		-1	-6	-5	
	1	6	5	-8	X

	1	7	11	-3	
3		3	30	123	
	1	10	41	120	X

	1	7	11	-3
-3		-3	-12	3
	1	4	-1	0

$$x^3+7x^2+11x-3=(x-(-3))\,(x^2+4x-1)$$

$$x^2+4x-1=0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-1)}}{2} = \frac{-4 \pm \sqrt{20}}{2}$$

$$= \frac{-4 \pm \sqrt{4} \cdot \sqrt{5}}{2} = \frac{-4 \pm 2\sqrt{5}}{2}$$

$$= -2 \pm \sqrt{5}$$

zeros: $\boxed{\{-3,\ -2+\sqrt{5},\ -2-\sqrt{5}\}}$