Test 1 online review

Sunday, September 24, 2017 6:49 PM

$f(x) = x(x-2)^2(x^2+1)(x-1)(x+1) = x(x-2)^2(x^2+1)(x-1)(x+1)$

Plot the above polynomial.

Find zeros: $x=0$

$x-2=0 \to x=2$

$x^2+1=0 \to x^2=-1$ ×

$x^2-1=0 \to x^2=1$

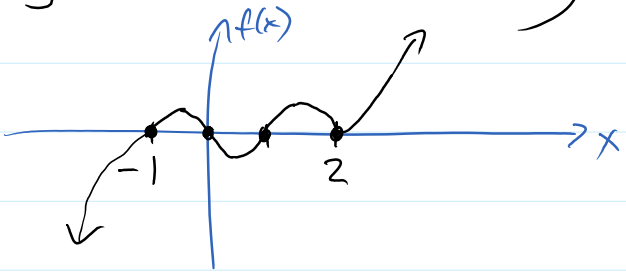
$x=\pm 1$

zeros	multip.
0	1
2	2
1	1
-1	1

degree of f is ~~5~~ 7

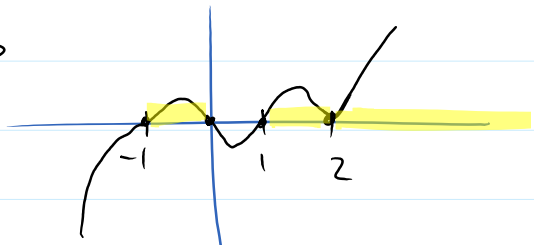
leading coefficient is 1

end behav:



Solve: $x(x-2)^2(x^2+1)(x-1)(x+1) > 0$

look for the interval where the function is above the x-axis



$$(-1,0) \cup (1,2) \cup (2,\infty)$$

Plot: $-3x^2(x-3)(x+4)^3(x^2+5)$

$x=0$

$x-3=0 \to x=3$

zero	multip.
0	2

$x=0$

$x-3=0 \to x=3$

$x+4=0 \to x=-4$

$x^2+5=0 \to x^2=-5 \to x=\pm\sqrt{-5}$

$x=\pm i\sqrt{5}$ complex number disregard it

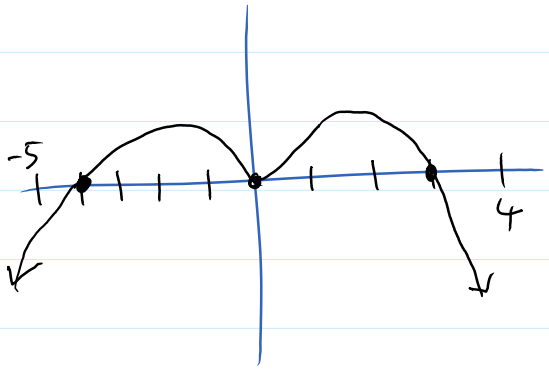
zero	multiplicity
0	2
3	1
-4	3

degree: $2+1+3+2$

$=8$

leading coeff: -3

end behav: ↙ ↘



Find the difference quotient for $f(x)$, where

$f(x) = 2x^2-3x$

$(a+b)^2 = a^2+2ab+b^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-3(x+h)-(2x^2-3x)}{h}$$

$$= \frac{2\cdot(x^2+2xh+h^2)-3x-3h-2x^2+3x}{h}$$

$$= \frac{2x^2+4xh+2h^2-3h-2x^2}{h} = \frac{h(4x+2h-3)}{h}$$

$$= \boxed{4x+2h-3}$$

$f(x) = \sqrt{2x-1}$

$(a-b)\cdot(a+b) = a^2-b^2$

$f(x) = \sqrt{2x-1}$

$(a - b) \cdot (a + b) = a^2 - b^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{2(x+h)-1} - \sqrt{2x-1}}{h} \cdot \frac{\sqrt{2(x+h)-1} + \sqrt{2x-1}}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}$$

$$= \frac{2(x+h)-1-(2x-1)}{h\left(\sqrt{2(x+h)-1} + \sqrt{2x-1}\right)} = \frac{2x+2h-1-2x+1}{h\left(\sqrt{2(x+h)-1} + \sqrt{2x-1}\right)}$$

$$= \frac{2h}{h\left(\sqrt{2(x+h)-1} + \sqrt{2x-1}\right)} = \boxed{\frac{2}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}}$$

$f(x) = \sqrt{x-2}$, $g(x) = \frac{1}{3x-1}$

$$(f \cdot g)(x) = \sqrt{x-2} \cdot \frac{1}{3x-1} = \boxed{\frac{\sqrt{x-2}}{3x-1}}$$

$$\frac{f(x)}{g(x)} = \frac{\sqrt{x-2}}{\frac{1}{3x-1}} = \frac{\frac{\sqrt{x-2}}{1}}{\frac{1}{3x-1}} = \frac{\sqrt{x-2}}{1} \cdot \frac{3x-1}{1} = \boxed{\sqrt{x-2} \cdot (3x-1)}$$

$$f(g(x)) = f\left(\frac{1}{3x-1}\right) = \boxed{\sqrt{\frac{1}{3x-1} - 2}} = \sqrt{\frac{1}{3x-1} - 2 \cdot \frac{3x-1}{3x-1}}$$

$$= \sqrt{\frac{1}{3x-1} - \frac{6x-2}{3x-1}} = \sqrt{\frac{1-6x+2}{3x-1}} = \sqrt{\frac{3-6x}{3x-1}}$$

What is a polynomial?

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

function	polynomial
$3x^2 - \frac{1}{3}x + \sqrt{2}$	yes
$\frac{2}{3}x^4 - 4x + 3 - \frac{1}{x}$	NO b/c $\frac{1}{x}$

Inverse function

- $f(x) = x^2 - 5 \rightarrow y = x^2 - 5$

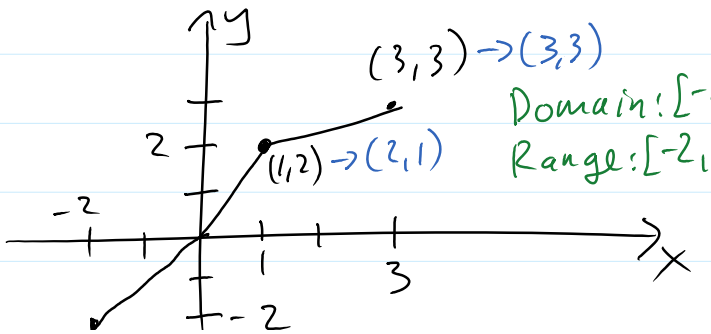
$$x = y^2 - 5$$
$$x + 5 = y^2$$
$$\pm\sqrt{x+5} = y$$

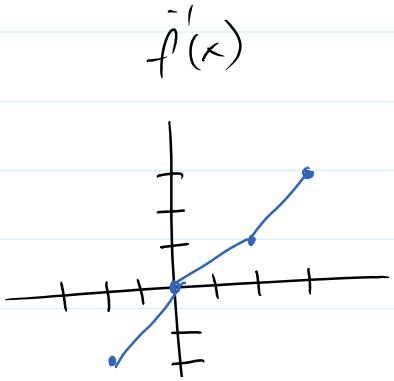
NO → the function is not one-to-one

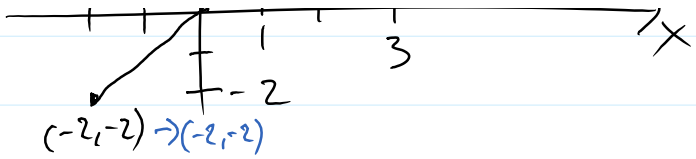
- $f(x) = \frac{1}{x-2} \rightarrow y = \frac{1}{x-2}$

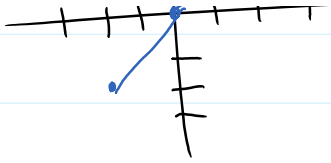
$$x = \frac{1}{y-2}$$
$$x(y-2) = 1$$
$$xy - 2x = 1$$
$$xy = 1 + 2x$$
$$y = \frac{1+2x}{x} \rightarrow f^{-1}(x) = \frac{1+2x}{x}$$

Find an inverse:









Find domain $f(x) = \dfrac{\sqrt{2-x}}{x+1}$

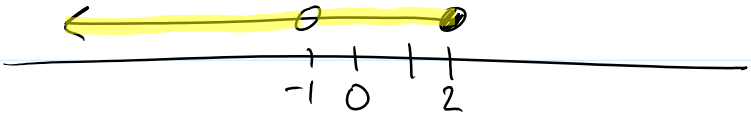
$2-x \geq 0$

$2 \geq x$

$x \leq 2$

$x+1 \neq 0$

$x \neq -1$



Domain: $(-\infty, -1) \cup (-1, 2]$

Domain: $f(x) = \dfrac{x+1}{\sqrt{2-x}}$

denominator $\neq 0$

$\sqrt{2-x} \neq 0$

$2-x \neq 0$

$2 \neq x$

$2-x \geq 0$

$2 \geq x$

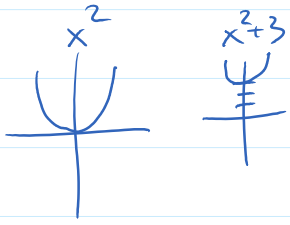
$(-\infty, 2)$

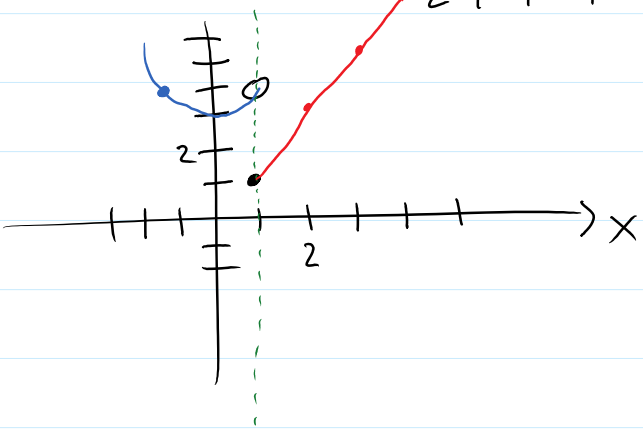
Graphing

$$f(x) = \begin{cases} x^2+3, & x < 1 \\ 2x-1, & x \geq 1 \end{cases}$$

$1^2+3 = 4$

$2 \cdot 1 - 1 = 1$



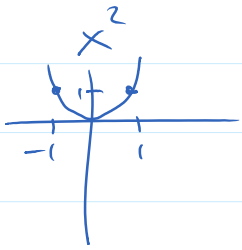


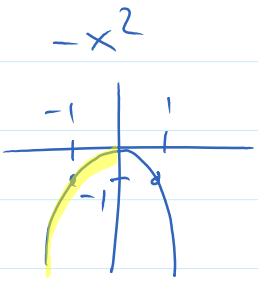
n(x) $\begin{cases} -x^2, & x < 0 \\ \end{cases}$ $-0 = 0$

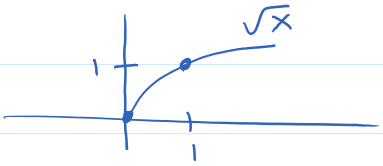
$$f(x)=\begin{cases} -x^2, & x \le 0 \\ \sqrt{x-2}, & x > 0 \end{cases}$$

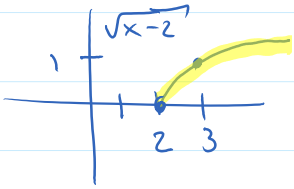
$-0=0$

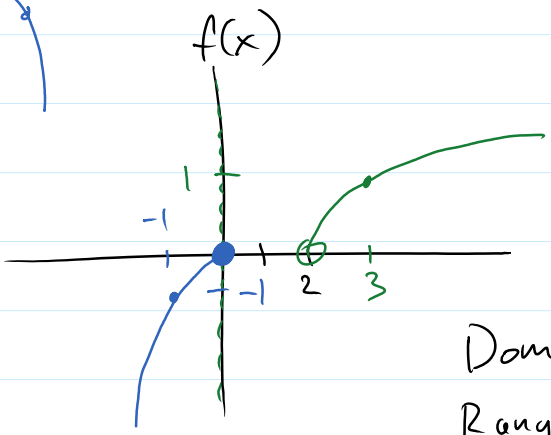
$\sqrt{0-2}=\sqrt{-2}$







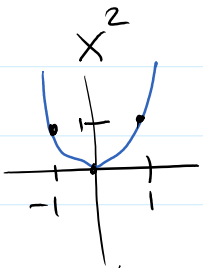




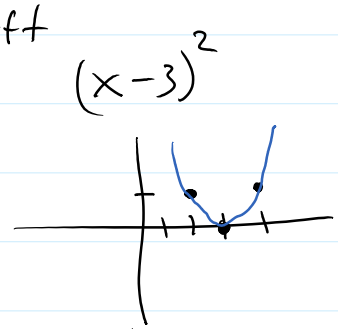
Domain: $(-\infty, 0] \cup (2, \infty)$

Range: $(-\infty, \infty)$

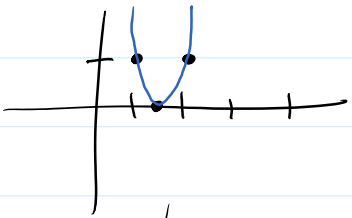
Graph $(2x-3)^2$



hor. shift by 3 →



hor. shink by factor of 2 →



x	y
−1	1
0	0
1	1

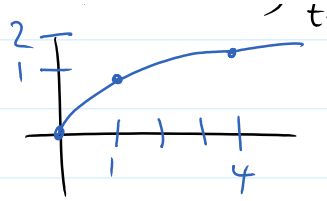
x	y
$-1+3=2$	1
$0+3=3$	0
$1+3=4$	1

x	y
$\frac{2}{2}=1$	1
$\frac{3}{2}=1.5$	0
$\frac{4}{2}=2$	1

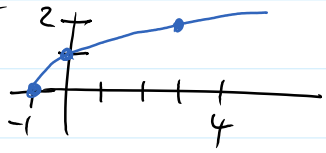
Graph $-3\sqrt{x+1}$

$\sqrt{x}$ → hor. shift to left by one → $\sqrt{x+1}$ → vertical stretch by 3 → $3\sqrt{x+1}$

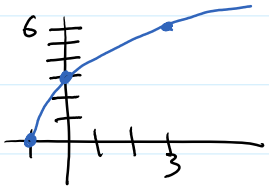
to left by 1 unit



x	y
0	0
1	1
4	2

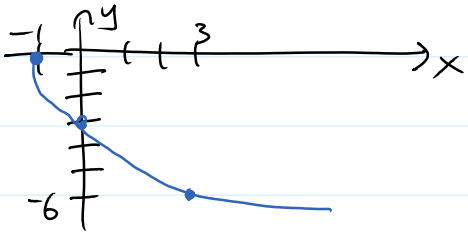


x	y
0-1=-1	0
1-1=0	1
4-1=3	2



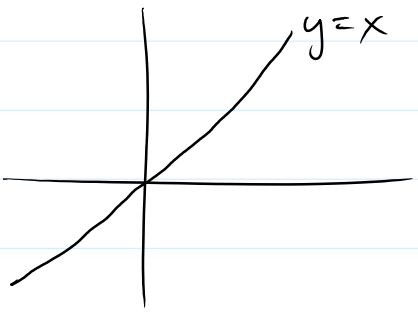
x	y
-1	0·3=0
0	1·3=3
3	2·3=6

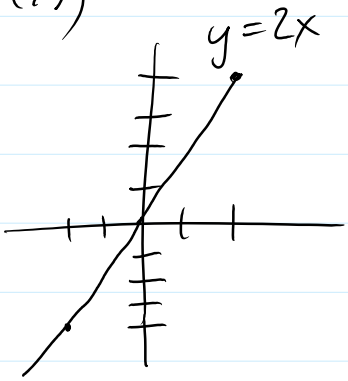
$-3\sqrt{x+1}$

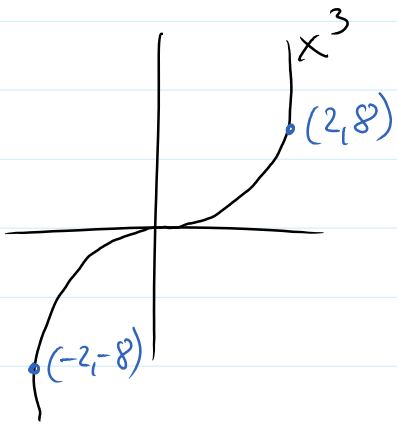


x	y
-1	-0
0	-3
3	-6

A graph is odd if it's reflection about the origin ($f(-x) = -f(x)$)







A graph is even if it's reflection about the y-axis

A graph is even if it's reflection about the y-axis

$(f(-x) = f(x))$