List all possible rational zeros:

$$f(x) = 6x^4 + 5x^3 - 7x^2 + 2$$

divisors: $\left\{ \begin{array}{l} 2: \pm 1, \pm 2 \\ 6: \pm 1, \pm 2, \pm 3, \pm 6 \end{array} \right\}$ $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 3, \pm 6}$

$$\pm 1, \pm 2, \pm \tfrac{1}{2}, \cancel{\pm \tfrac{2}{2}} = 1, \pm \tfrac{1}{3}, \pm \tfrac{2}{3}, \pm \tfrac{1}{6}, \cancel{\pm \tfrac{2}{6}} = \tfrac{1}{3}$$

Solutions $\boxed{\left\{ \pm 1, \pm 2, \pm \tfrac{1}{2}, \pm \tfrac{1}{3}, \pm \tfrac{2}{3}, \pm \tfrac{1}{6} \right\}}$

10) Use synth. division : $(x^5 + x^3 - 5) \div (x - 2)$

$$1x^5 + 0x^4 + 1x^3 + 0x^2 + 0x - 5$$

	1	0	1	0	0	−5
2	↓	2	4	10	20	40
	1	2	5	10	20	35 = remaindr

$$\underbrace{1 \quad 2 \quad 5 \quad 10 \quad 20}_{x^4 + 2x^3 + 5x^2 + 10x + 20}$$

$$(x^5+x^3-5)\div(x-2) = x^4+2x^3+5x^2+10x+20+\frac{35}{(x-2)}$$

3.4

Find a polynomial so that
3, $2-i$ are zeros and $f(1)=42$

Zeros: 3, $2-i$, $\underbrace{2+i}_{\text{complex conjugate of } 2-i}$

$3-2i \to 3+2i$
$-4i \to +4i$
$-i \to i$

$$f(x) = A\cdot(x-3)(x-(2-i))(x-(2+i))$$
$$= A\cdot(x-3)(\underbrace{x-2}+i)(\underbrace{x-2}-i)$$
$$(a+b)(a-b) = a^2-b^2$$
$$= A\,(x-3)\left((x-2)^2-(i)^2\right)$$
$$= A\,(x-3)\left(x^2-2\cdot x\cdot 2+4-(-1)\right)$$
$$= A\,(x-3)(x^2-4x+5)$$
$$= A\,(x^3-4x^2+5x-3x^2+12x-15)$$
$$= A\,(x^3-7x^2+17x-15)$$

↓ solve for A

$$f(1)=42$$
$$A(1^3-7\cdot1^2+17\cdot1-15)=42$$
$$A(11-15)=42$$
$$A\cdot(-4)=42$$
$$A=\frac{42}{-4}=\boxed{-\frac{21}{2}}$$

$$f(x) = -\frac{21}{2}\left(x^3 - 7x^2 + 17x - 15\right)$$
$$= \boxed{-\frac{21}{2}x^3 + \frac{21}{2}\cdot 7x^2 - \frac{21}{2}\cdot 17x + \frac{21}{2}\cdot 15}$$

Sketch: $\frac{4-x^2}{x^2-3x-4} = \frac{(2-x)(2+x)}{(x-4)(x+1)}$

Domain:

$(x-4)(x+1) = 0$

$x-4=0 \qquad x+1=0$

$x=4 \qquad x=-1$

$\boxed{(-\infty,-1)\cup(-1,4)\cup(4,\infty)}$

asymp:

simplify: $\frac{(2-x)(2+x)}{(x-4)(x+1)}$

vertical

$(x-4)(x+1)=0$

$\boxed{x=4,-1} \Leftrightarrow \boxed{\begin{array}{l} x=4 \\ x=-1 \end{array}}$

hor./slant asym

hor. asym. $y = \frac{-1}{1} \to \boxed{y=-1}$

Symmetry

$$f(-x) = \frac{4-(-x)^2}{(-x)^2-3(-x)-4} = \frac{4-x^2}{x^2+3x-4} \neq f(x)$$

not even

$$-\frac{4-x^2}{\quad} = -\frac{4-x^2}{\quad} \neq -f(x)$$

$$= \frac{4-x^2}{-(-x^2-3x+4)} = -\frac{4-x^2}{-x^2-3x+4} \neq -f(x)$$

not odd

$$g(x) = \frac{x}{x^3-x}$$

$$g(-x) = \frac{-x}{(-x)^3-(-x)}$$

$$= \frac{-x}{-x^3+x} = \frac{+x}{+(x^3-x)}$$

$$= \frac{x}{x^3-x} = \underbrace{g(x)}_{\text{even}}$$

$$f(x) = \frac{x^3}{x^2-3}$$

$$f(-x) = \frac{(-x)^3}{(-x)^2-3} = \frac{-x^3}{x^2-3}$$

$$= -\frac{x^3}{x^2-3} = \underbrace{-f(x)}_{f(x)\text{ is odd}}$$

$$f(x) = \frac{2}{x^3-3x}$$

$$f(-x) = \frac{2}{(-x)^3-3(-x)}$$

$$= \frac{2}{-x^3+3x} = \frac{2}{-(x^3-3x)}$$

$$= -\frac{2}{x^3-3x} = \underbrace{-f(x)}_{\text{odd}}$$

y-int

$$f(0) = \frac{4-0}{0^2-3\cdot 0-4} = \frac{4}{-4} = -1$$

$$\boxed{(0, -1)}$$

x-int

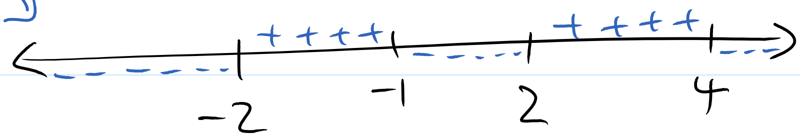
$$f(x) = \frac{(2-x)(2+x)}{(x-4)(x+1)}$$

$$(2-x)(2+x) = 0$$

$$x = \pm 2$$

$\boxed{(-2,0),\ (2,0)}$

x-int.
ver. asym.



$$f(x)=\frac{(2-x)(2+x)}{(x-4)(x+1)}$$

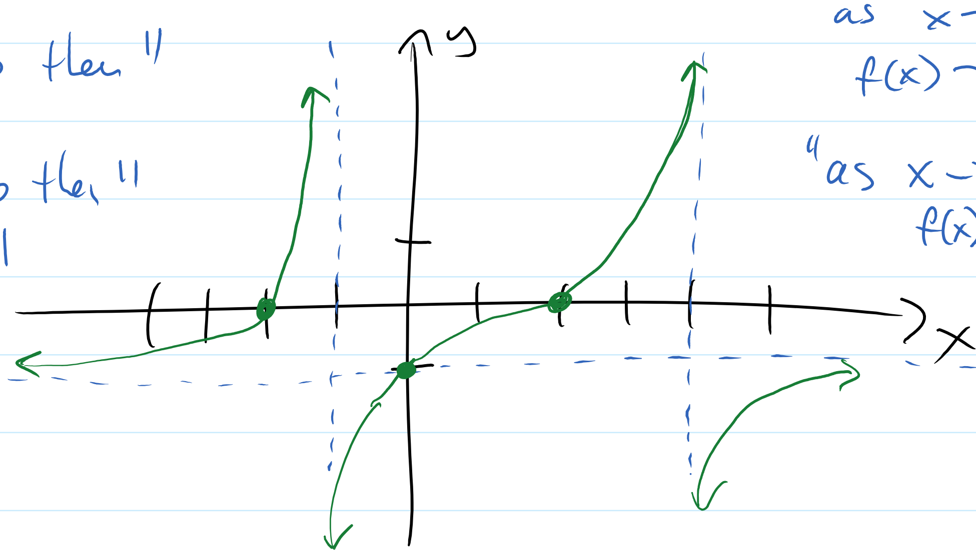
	$(-\infty,-2)$ -3	$(-2,-1)$ -1.5	$(-1,2)$ 0	$(2,4)$ 3	$(4,\infty)$ 5
$(2-x)$	+	+	+	−	−
$(2+x)$	−	+	+	+	+
$(x-4)$	−	−	−	−	+
$(x+1)$	−	−	+	+	+
sign of $f(x)$	−	+	−	+	−

$f(x)\geq 0$
$[-2,-1)\cup[2,4)$

$f(x)\leq 0$
$(-\infty,-2]\cup(-1,2]\cup(4,\infty)$

"as $x\to\infty$ then"
$f(x)\to -1$

"as $x\to-\infty$ then"
$f(x)\to -1$



"as $x\to 4^-$ then"
$f(x)\to\infty$

"as $x\to -1^+$ then"
$f(x)\to-\infty$

Crossing with the hor. asym:

$$\frac{4-x^2}{\underbrace{(x-4)(x+1)}_{x^2-3x-4}}=-1$$

$$4-x^2=(-1)(x^2-3x-4)$$

$$4-x^2=(-1)(x^2-3x-4)$$

$$4-\cancel{x^2}=-\cancel{x^2}+3x+4$$

$$-4 \qquad\qquad -4$$

$$0=3x$$

$$\boxed{x=0}$$

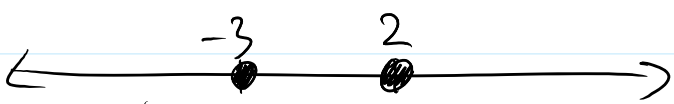
Find the domain: $f(x)=\sqrt{(x-2)^2(x+3)}$

Solve: $(x-2)^2(x+3)\geq 0$

Find: $(x-2)^2(x+3)=0$

$x-2=0$ $\qquad$ $x+3=0$

$x=2$ $\qquad$ $x=-3$



	$(-\infty,-3]$ -4	$[-3,2]$ 0	$[2,\infty)$ 3
$(x-2)^2$	+	+	+
$x+3$	−	+	+
	−	$\boxed{+}$	$\boxed{+}$

Domain: $[-3,2]\cup[2,\infty)=\boxed{[-3,\infty)}$