

Solve:

$$\bullet 5^{7x+2} = 2^{x-5}$$

$$5^{7x} \cdot 5^2 = \frac{2^x}{2^5}$$

$$5^{7x} \cdot 25 = \frac{2^x}{2^5} \quad // \ln$$

$$\ln(5^{7x} \cdot 25) = \ln\left(\frac{2^x}{2^5}\right)$$

$$\ln(5^{7x}) + \ln 25 = \ln 2^x - \ln 2^5$$

$$7x \ln 5 + \ln 25 = x \ln 2 - 5 \ln 2$$

$$7x \ln 5 - x \ln 2 = -\ln 25 - 5 \ln 2$$

$$x(7 \ln 5 - \ln 2) = -2 \ln 5 - 5 \ln 2$$

$$x = \frac{-2 \ln 5 - 5 \ln 2}{7 \ln 5 - \ln 2} = \frac{2 \ln 5 + 5 \ln 2}{-7 \ln 5 + \ln 2}$$

$$\frac{2}{2} = \frac{-2}{-2}$$

$$\bullet e^{4x} - 7e^{2x} - 44 = 0$$

$$e^{2 \cdot 2x} - 7e^{2x} - 44 = 0$$

$$(e^{2x})^2 - 7(e^{2x}) - 44 = 0$$

$$\underline{u = e^{2x}}$$

..? ... u ->

Rule:

$$x^{a+b} = x^a \cdot x^b$$

$$x^{a-b} = \frac{x^a}{x^b}$$

// ln

$$\rightarrow \ln(5^{7x+2}) = \ln(2^{x-5})$$

$$(7x+2) \ln 5 = (x-5) \ln 2$$

$$\underline{u = e}$$

$$u^2 - 7u - 44 = 0$$

$$(u+4)(u-11) = 0$$

$$u = 4$$

$$e^{2x} = -4$$

$$\ln e^{2x} = \ln -4$$

$$2x \ln e = \ln(-4)$$

$$2x = \ln(-4)$$

none

Does
not
exist

$$x = \frac{\ln 11}{2}$$

$$u = 11$$

$$e^{2x} = 11$$

$$2x = \ln 11$$

$$x = \frac{\ln 11}{2}$$

$$\bullet 3^{1-5x} = 5^x$$

$$\ln 3^{1-5x} = \ln 5^x$$

$$(1-5x) \ln 3 = x \ln 5$$

$$\ln 3 - 5x \ln 3 = x \ln 5$$

$$-x \ln 5 - 5x \ln 3 = -\ln 3$$

$$x(-\ln 5 - 5 \ln 3) = -\ln 3$$

$$x = \frac{-\ln 3}{-\ln 5 - 5 \ln 3} \cdot \frac{-1}{-1} = \frac{\ln 3}{\ln 5 + 5 \ln 3}$$

$$\bullet 7^{2x} + 7^x - 42 = 0$$

$$\bullet \quad 7^{2x} + 7^x - 42 = 0$$

$$(7^x)^2 + 7^x - 42 = 0$$

$$u^2 + u - 42 = 0 \quad \underline{u = 7^x}$$

$$(u-6)(u+7) = 0$$

$$u-6=0$$

$$u=6$$

$$7^x = 6$$

$$x \ln 7 = \ln 6$$

$$x = \frac{\ln 6}{\ln 7} = \boxed{\log_7 6}$$

$$u+7=0$$

$$u=-7$$

$$7^x = -7$$

$$x \ln 7 = \ln -7$$

none

$$\text{Simplify: } 10^{\log 30 - \log 5} = \frac{10^{\log 30}}{10^{\log 5}}$$

$$= \frac{10^{\log_{10} 30}}{10^{\log_{10} 5}}$$

$$= \frac{30}{5} = \boxed{6}$$

$$\boxed{b^{\log_b x} = x}$$

Solve:

$$\bullet \quad 5e^{5x} = 2195$$

$$e^{5x} = \frac{2195}{5}$$

$$c = 5$$

$$e^{5x} = 439 \quad // \ln$$

$$\ln e^{5x} = \ln 439$$

$$5x = \ln 439$$

$$x = \boxed{\frac{\ln 439}{5}}$$

$$\bullet \quad 3^{2x} + 3^{x+1} - 40 = 0$$

$$(3^x)^2 + 3^{x+1} - 40 = 0$$

$$(3^x)^2 + 3^x \cdot 3^1 - 40 = 0$$

$$(3^x)^2 + 3 \cdot (3^x) - 40 = 0$$

$$u = 3^x$$

$$u^2 + 3u - 40 = 0$$

⋮

Simplify to one log:

$$2 \ln(5x+3) - 5 \ln(x-3) - 4 \ln x$$

$$= \ln[(5x+3)^2] - \ln[(x-3)^5] - \ln x^4$$

$$\ln A + \ln B = \ln AB$$

$$\ln A - \ln B = \ln \frac{A}{B}$$

$$= \ln \left[\frac{(5x+3)^2}{(x-3)^5} \right] - \ln x^4$$

$$= \ln \left[\frac{(5x+3)^2}{(x-3)^5} \cdot \frac{1}{x^4} \right] = \ln \left[\frac{(5x+3)^2}{(x-3)^5 x^4} \right]$$

$$= \ln \left[\frac{(5x+3)^2}{x^4 (x-3)^5} \right]$$

Solve:

$$\ln(x-6) + \ln(x+1) = \ln(x-15)$$

$$\ln((x-6) \cdot (x+1)) = \ln(x-15)$$

$$\ln(x^2 - 5x - 6) = \ln(x-15)$$

↓

$$x^2 - 5x - 6 = x - 15$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)(x-3) = 0$$

$x=3$ \rightarrow test: $\ln(3-6) + \ln(3+1) = \ln(3-15)$
 $\ln(-3) + \ln(4) = \ln(-12)$

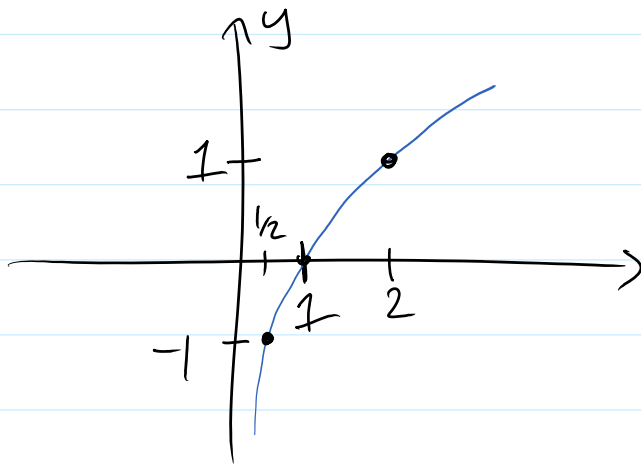
not a solution

b/c -3 is not in the domain of $\ln(x)$

There is no solution

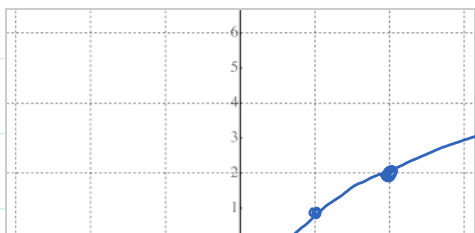
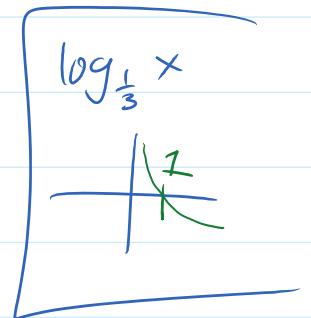
Plot: $y = \log_2(-x+2)$

$\log_2(x)$ $\xrightarrow{\text{hor. shift left by 2}}$ $\log_2(x+2)$ $\xrightarrow{\text{reflection about the y-axis}}$ $\log_2(-x+2)$

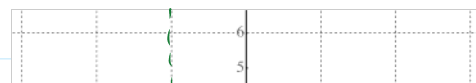


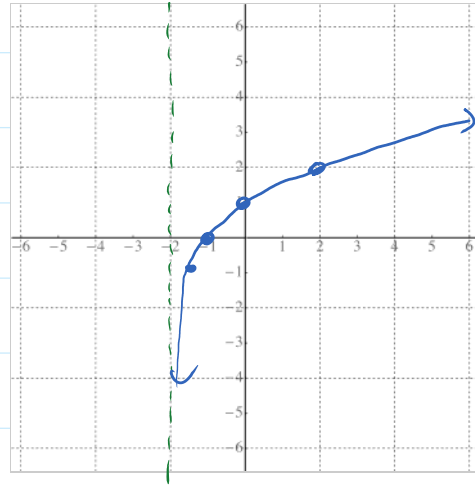
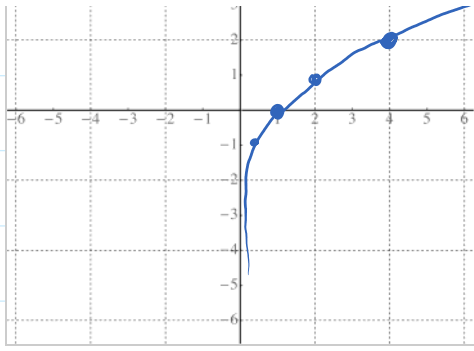
$$\begin{cases} \log_b b = 1 \\ \log_2 2 = 1 \\ \log_b \frac{1}{b} = \log_b b^{-1} = -1 \\ \log_2 \frac{1}{2} = -1 \end{cases}$$

$y = \log_2 X$

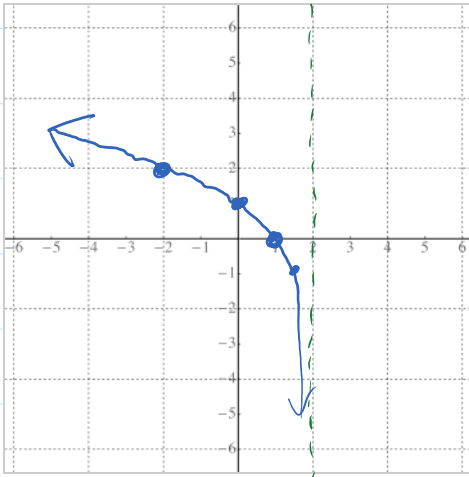


$y = \log_2(x+2)$





$$y = \log_2(-x+2)$$



Solve:

$$\bullet 9(4^{7x}) = 2$$

$$\frac{9 \cdot 4^{7x}}{9} = \frac{2}{9}$$

$$4^{7x} = 2/9$$

⋮

$$\bullet 5^{x-2} = 4^{2x+3}$$

$$(x-2) \ln 5 = (2x+3) \ln 4$$

⋮

Find domain of $\log(x^4 + x^3 - x^2 + x - 2)$

need to solve: $x^4 + x^3 - x^2 + x - 2 > 0$

