Diagnostic test

Name keg

1. Evaluate each expression without using a calculator

   a) $(\frac{2}{3})^{-2} = \frac{3^2}{2^2} = \frac{9}{4} = 2.25$

   b) $16^{-3/4}$ $(2^4)^{-\frac{3}{4}} = 2^{-\frac{4\cdot 3}{4}} = 2^{-3} = \boxed{\frac{1}{8}}$

2. Simplify

$$\left(\frac{2x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2} = \left(2x^{\frac{3}{2}-2}y^{3+\frac{1}{2}}\right)^{-2}$$

$$= \left(2x^{-\frac{1}{2}}y^{\frac{7}{2}}\right)^{-2} = \frac{1}{2^2}\cdot X^{-\frac{1}{2}\cdot(-2)}\cdot y^{\frac{7}{2}\cdot(-2)}$$

$$= \boxed{\frac{1}{4}x\cdot y^{-7}} = \frac{1\cdot x}{4y^7} = \frac{x}{4y^7}$$

3. Expand and simplify

a)

$$(x+3)(4x-5)$$

$= 4x^2 + 12x - 5x - 15 = 4x^2 + 7x - 15$

b)

$$(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$$

$\boxed{a - b}$

4. Factor the expression

a)

$$2x^2+5x-12$$

$(2x-3)(x+4)$

b)

$$4x^2-25$$

$(2x-5)(2x+5)$

5. Simplify

a)

$$\frac{\frac{y}{x}-\frac{x}{y}}{\frac{1}{y}-\frac{1}{x}} = \frac{\frac{y^2-x^2}{xy}}{\frac{x-y}{xy}} = \frac{y^2-x^2}{\cancel{xy}}\cdot\frac{\cancel{xy}}{x-y}$$

$$= \frac{-(\cancel{x-y})(x+y)}{\cancel{x-y}} = \boxed{\cancel{-x+y}}$$

$$\boxed{-x-y}$$

b)

$$\frac{x^2}{x^2-4}-\frac{x+1}{x+2} = \frac{x^2}{(x-2)(x+2)} - \frac{x+1}{x+2}$$

$$= \frac{x^2-(x+1)(x-2)}{(x-2)(x+2)} = \frac{x^2-(x^2-x-2)}{(x-2)(x+2)}$$

$$= \frac{\cancel{x+2}}{(x-2)\cancel{(x+2)}} = \boxed{\frac{1}{x-2}}$$

6. Rationalize the expression and simplify

a)

$$\frac{\sqrt{10}}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{10}(\sqrt{5}+2)}{5-4}$$

$$= \boxed{\sqrt{10}(\sqrt{5}+2)}$$

b)

$$\frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

$$= \frac{\cancel{4}+h-\cancel{4}}{h(\sqrt{4+h}+2)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)} = \boxed{\frac{1}{\sqrt{4+h}+2}}$$

7. Find an equation of the line that contains the points (-3,2) and (1,1).

slope: $\frac{1-2}{1-(-3)} = \frac{-1}{4}$

$y = \frac{-1}{4}x + b$

plug in (1,1): ——— OR ——— use:

$1 = \frac{-1}{4} + b \rightarrow b = 1 + \frac{1}{4} = \frac{5}{4}$

$$\boxed{y = \frac{-1}{4}x + \frac{5}{4}}$$

$y - y_0 = m(x - x_0)$

$(x_0, y_0) = (1,1)$:

$y - 1 = \frac{-1}{4}(x-1)$

$y = \frac{-1}{4}x + \frac{1}{4} + 1$

$$\boxed{y = \frac{-1}{4}x + \frac{5}{4}}$$

8. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

$$= \frac{(2+h)^3 - 8}{h} = \frac{\cancel{8} + 3\cdot 2^2 h + 3\cdot 2\cdot h^2 + h^3 - \cancel{8}}{h} = \frac{\cancel{h}(3\cdot 4 + 6\cdot h + h^2)}{\cancel{h}}$$

$$= \boxed{12 + 6h + h^2}$$

9. Find the domain of the function

a)

$$\frac{\sqrt[3]{x}}{x^2+1}$$

denom: $x^2 + 1 = 0$

$x^2 = -1$

No sol.

$\sqrt[3]{x}$ is def for all real num.

$$\boxed{D = \mathbb{R}.}$$

b)

$$\sqrt{4-x} + \sqrt{x^2-1}$$

$\sqrt{4-x}$: $4 - x \geq 0$

$4 \geq x$

$\sqrt{x^2-1}$: $x^2 - 1 \geq 0$

$x^2 \geq 1$

$|x| \geq 1$

$x \geq 1$ or $x \leq -1$

$$\boxed{(-\infty, -1] \cup [1, 4]}$$

10. Find the minimum value of the function $f(x) = \frac{\cos 3x}{2}$. (no justification necessary)

$\boxed{-\frac{1}{2}}$

Give at least one value of x where the minimum value of f is attained.

$\cos(3x) = -1$ if $3x = \pi \Rightarrow x = \frac{\pi}{3}$

Thus $\frac{\cos(3 \cdot \frac{\pi}{3})}{2} = -1 \Rightarrow \boxed{x = \frac{\pi}{3}}$ $\left(+ \frac{2}{3} k \cdot \pi\right)$

11. Simplify

$$\frac{1}{\sqrt[5]{x^2}} x^2$$

$x^{-\frac{2}{5}} \cdot x^2 = x^{-\frac{2}{5}+2} = x^{\frac{-2+10}{5}} = \underline{x^{\frac{8}{5}}} = \underline{\sqrt[5]{x^8}} = \underline{x \cdot \sqrt[5]{x^3}}$

12. Are the following statements true?

a) $\sqrt{x^2 + y^2} = x + y$

<u>NO</u>

b) $(a + b)^{-1} = a^{-1} + b^{-1}$

<u>NO</u>

c) $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$

<u>NO</u>