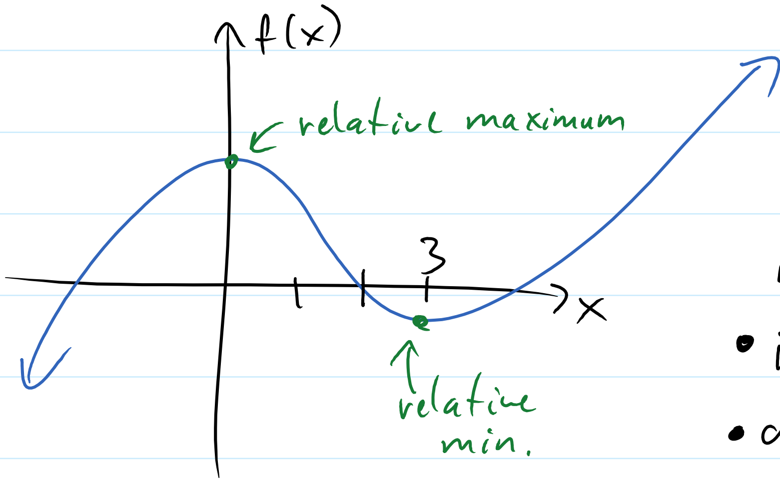
10/02

Monday, October 2, 2017 9:50 AM

HW 7, 8, 9 (2.5, 3.1, 3.2) due on Sunday, 10/8.
In class Quiz on Monday 10/9

Section 3.1



Find the intervals, where the function is increasing/decreasing.

- increasing on $(-\infty, 0), (3, \infty)$
- decreasing on $(0, 3)$

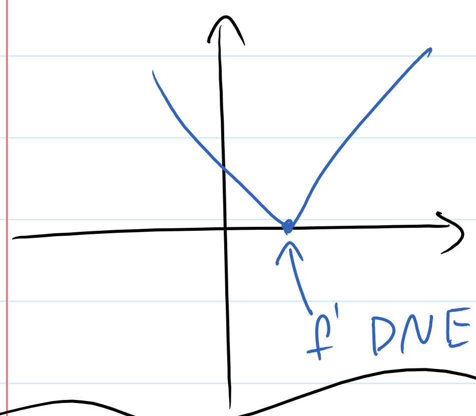
We can also look at the derivative and find: inc $\Leftrightarrow f'(x) > 0$

dec $\Leftrightarrow f'(x) < 0$

Def: The graph of $f(x)$ is said to have a relative maximum (minimum) at $x = c$, if $f(c) \geq f(x)$ ($f(c) \leq f(x)$) for all x in some interval $a < x < b$ containing c.

Def: A number c in the domain of $f(x)$ is a critical number if either $f'(c) = 0$ or

$f'(c)$ DNE.



Dom: $[0,\infty)$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \dfrac{1}{2\sqrt{x}}$

$f'(0)$ DNE

__Ex:__ Find all critical numbers of:

$$f(x) = 2x^4 - 4x^2 + 3$$

$$f'(x) = 8x^3 - 8x = 0$$

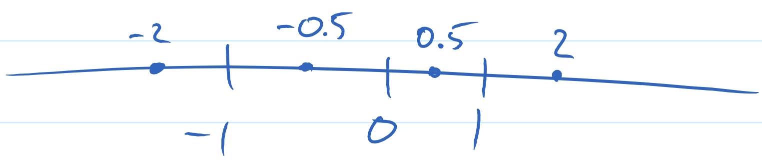
$$8x(x^2-1) = 0$$

$$8x(x-1)(x+1) = 0$$

$x=0$ $\quad x=1$ $\quad x=-1$

$$\boxed{0, \pm 1}$$

Find the intervals of inc/dec.



Plug $-2, -0.5, 0.5$ and 2 into $f'(x)$ to see if positive (f is increasing) or negative (f is dec)

$f'(-2) = -16(-3)(-1) = -48 < 0$ __dec__

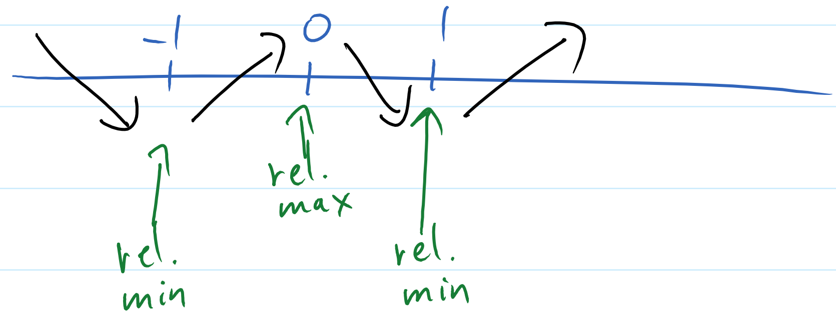
$f'(-.5) = -4(-1.5)(0.5) > 0$ inc

$f'(-2) = -16(-3)(-1) = -48 < 0$ dec

$f'(-.5) = -4(-1.5)(0.5) > 0$ inc

$f'(0.5) = + \; - \; + = < 0$ dec

$f'(2) = + \; + \; + > 0$ inc



Thm: (The First Derivative test)

Let c be a critical number for $f(x)$.
The critical point $(c, f(c))$ is

- rel. maximum if f' changes from $+$ to $-$, at c
- rel minimum if f' changes from $-$ to $+$, at c.

Ex: Find int. of inc/dec. and relative min/max

$$f(x) = x^4 + 8x^3 + 18x^2 - 8$$

Dom: $(-\infty, \infty)$

$$f'(x) = 4x^3 + 24x^2 + 36x = 0$$

$$f'(x) = 4x^3 + 24x^2 + 36x = 0$$

$$4x(x^2 + 6x + 9) = 0$$

$$4x(x+3)(x+3) = 0$$

$x = 0, -3$ Crit. numbers: $0, -3$

-3 0

$f'(x)$	$(-\infty,-3)$	$(-3,0)$	$(0,\infty)$
	-4	-1	1
$4x$	$-$	$-$	$+$
$(x+3)^2$	$+$	$+$	$+$
sign of f'	$-$	$-$	$+$
	$\searrow$	$\searrow$	$\nearrow$

f is
- inc on $(0,\infty)$
- dec on $(-\infty,-3), (-3,0)$

- f has relative min at $x=0$, the relative minimum is $f(0) = -8$.
- Also, the point $(0,-8)$ is rel. min.

(on exam/quiz, the x-value is enough)

Ex: HW: $f(t) = \dfrac{t}{t^2+3}$ find rel. min/max.

Dom: $t^2 + 3 = 0$

$t^2 = -3$ X

$t^2 = -3$ X

$(-\infty, \infty)$

$$f'(t) = \frac{t^2+3 - t\cdot 2t}{(t^2+3)^2} = 0$$

$$\frac{3-t^2}{(t^2+3)^2} = 0$$

$\underbrace{f'(t)=0}_{3-t^2=0}$

$\underbrace{f' \text{ DNE}}_{(t^2+3)^2=0}$