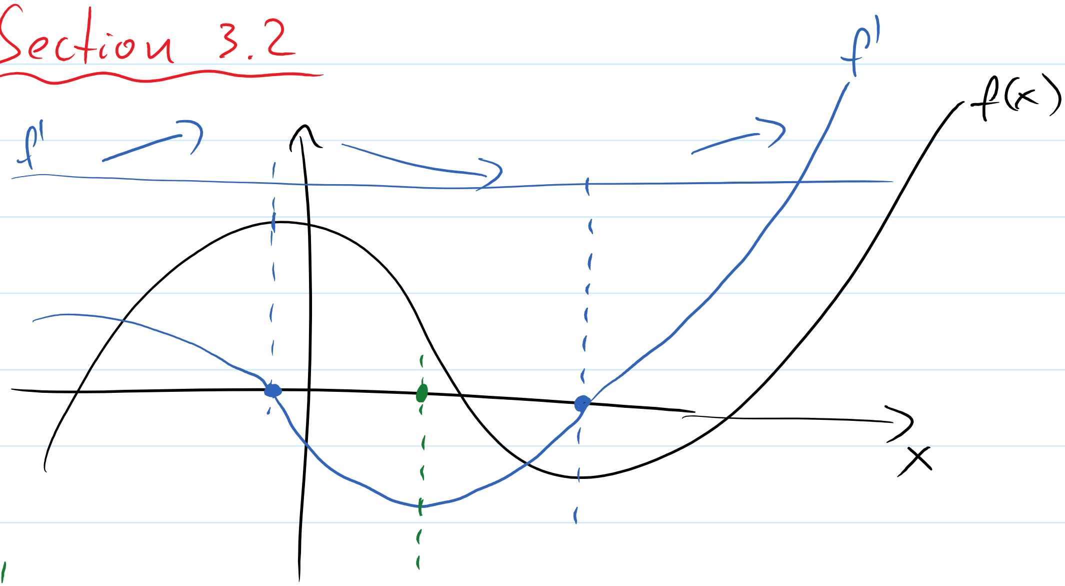
Section 3.2



f''

negative | positive

"concave down" | "concave up"

Def: If the function $f(x)$ is differentiable on (a,b), the graph of f is concave down (up) on (a,b) if

$f'(x)$ is decreasing $\Leftrightarrow f''(x)$ is negative on (a,b)

$f'(x)$ is increasing $\Leftrightarrow f''(x)$ is positive on (a,b)

Ex: Find the intervals of concavity:

$$f(x) = 2x^6 - 5x^4 + 7x - 3$$

$$f'(x) = 12x^5 - 20x^3 + 7$$
$$f''(x) = 60x^4 - 60x^2 = 0$$
$$60x^2(x^2 - 1) = 0$$

$$60x^2(x-1)(x+1)=0$$
$$x^2=0 \qquad x=\pm 1$$
$$x=0$$

f''	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
$60x^2$	+	+	+	+
$x-1$	−	−	−	+
$x+1$	−	+	+	+
f''	+ ∪	− ∩	− ∩	+ ∪

inflection pts.

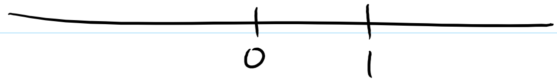
Def: $x=c$ is an inflection point of $f(x)$ if c is in the domain of f and $f''(x)$ changes its sign at c. [point $(c, f(c))$]

Ex: Find inflection pts:

- $f(x)=3x^5-5x^4-1$

$$f'(x)=15x^4-20x^3$$
$$f''(x)=60x^3-60x^2=0$$
$$60x^2(x-1)=0$$
$$x=0,1$$

$x = 0, 1$



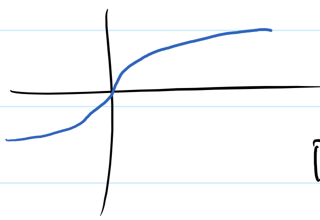
f''	$(-\infty, 0)$ $\;-1$	$(0, 1)$ $\;\frac{1}{2}$	$(1, \infty)$ $\;2$
$60x^2$	$+$	$+$	$+$
$(x-1)$	$-$	$-$	$+$
f''	$-$	$-$	$+$
	$\frown$	$\frown$	$\smile$

$x=1$ is an inflection pt

$$f(1) = 3\cdot 1^5 - 5\cdot 1^4 - 1 = -3$$

$$\boxed{(1, -3)}$$

- $g(x) = \sqrt[3]{x}$
  $= x^{\frac{1}{3}}$

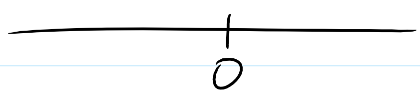


Dom: $(-\infty, \infty)$

$$g'(x) = \frac{1}{3} x^{-2/3}$$

$$g''(x) = \frac{1}{3}\cdot\left(-\frac{2}{3}\right) x^{-5/3} = \underbrace{\frac{-2}{9}\, x}_{} {}^{-5/3} = \frac{-2}{9x^{5/3}} = \frac{-2}{9x\cdot\sqrt[3]{x^2}}$$

$x = 0$ } do not mind the exponent



g''	$(-\infty, 0)$ $\;-1$	$(0, \infty)$ $\;1$
$-\frac{2}{9}$	$-$	$-$
$x^{-5/3}$	$-$	$+$

$x^{-5/3}$ $\frac{1}{x^{5/3}}$	$-$	$+$
	$+$	$-$

$\boxed{x=0}$ — inflection pt.

$f(0) = \sqrt[3]{0} = 0$

$\boxed{(0,0)}$

<u>Ex:</u> $f(x) = 3x^4 - 2x^3 - 12x^2 + 18x + 15$,

Find intervals inc/dec, concave up/down; relative extrema, inflection points.

<u>Dom:</u> $(-\infty, \infty)$

$f'(x) = 12x^3 - 6x^2 - 24x + 18$

$f''(x) = 36x^2 - 12x - 24$

$$f'(x) = 0$$
$$12x^3 - 6x^2 - 24x + 18 = 0$$
$$6(2x^3 - x^2 - 4x + 3) = 0$$

"rational zeros thm"

$x=1$ is a root, let's do synt. division:

	2	−1	−4	3
1	↓	2	1	−3
	2	1	−3	0

$$6(x-1)(2x^2 + x - 3) = 0$$
$$6(x-1)(2x+3)(x-1) = 0$$

$x = 1, -\frac{3}{2}, 1$

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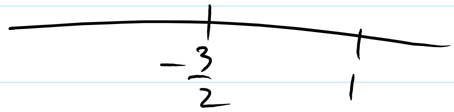


f'	$(-\infty, -\frac{3}{2})$	$(-\frac{3}{2}, 1)$	$(1, \infty)$
$6(2x+3)$	$-$ -2	$+$ -1	$+$ 2
$(x-1)^2$	$+$	$+$	$+$
	$-$ ↘	$+$ ↗	$+$ ↗

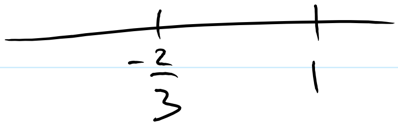
$x = \frac{-3}{2} \Leftrightarrow$ rel. min

$f(-\frac{3}{2}) = -17.06$

$\boxed{(-\frac{3}{2}, -17.06)}$

Concavity:

$$f''(x) = 36x^2 - 12x - 24 = 0$$
$$12(3x^2 - x - 2) = 0$$
$$12(3x+2)(x-1) = 0$$
$$x = -\frac{2}{3}, 1$$



f''	$(-\infty, -\frac{2}{3})$	$(-\frac{2}{3}, 1)$	$(1, \infty)$
$12(3x+2)$	$-$ -2	$+$ 0	$+$ 2
$x-1$	$-$	$-$	$+$
	$+$ ∪	$-$ ∩	$+$ ∪

$x = -\frac{2}{3}$, $x = 1$ } Inflection points

$f(-\frac{2}{3}) = -1.15$

$f(1) = 22$

Inflection pts: $\boxed{(-\frac{2}{3}, -1.15), (1, 22)}$

Inflection pts: $\boxed{\left(-\frac{2}{3}, -1.15\right), (1, 22)}$