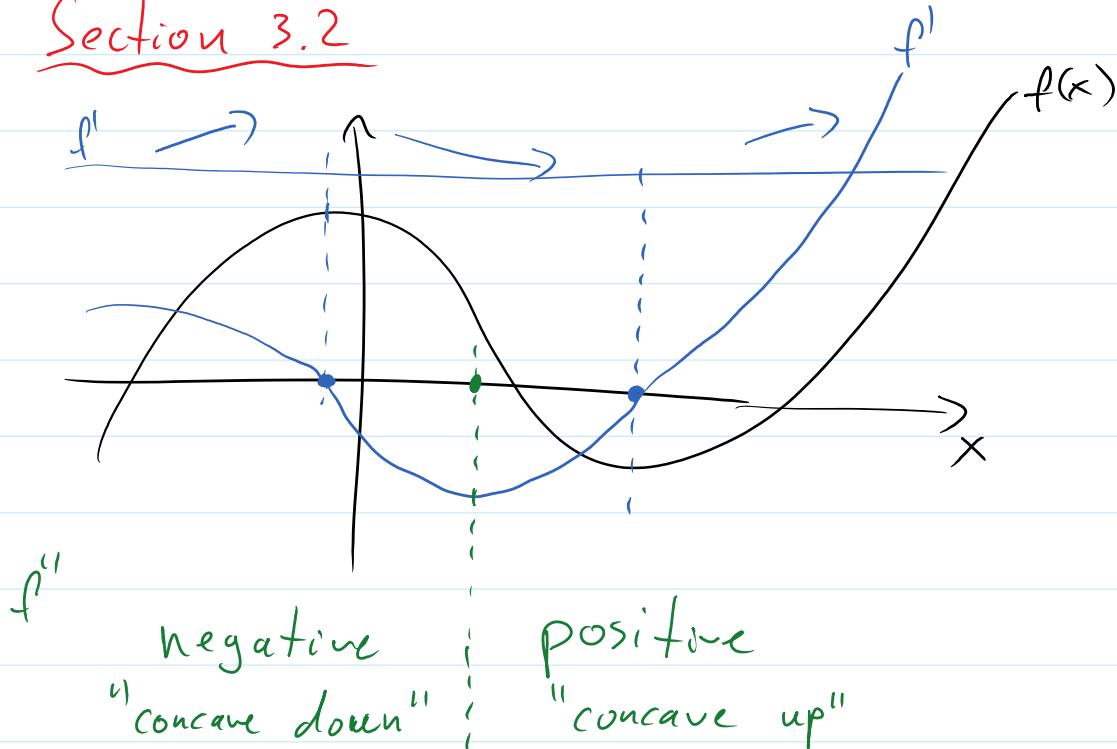


## Section 3.2



Def: If the function  $f(x)$  is differentiable on  $(a,b)$ , the graph of  $f$  is concave down (up) on  $(a,b)$  if  $f'(x)$  is decreasing  $\Leftrightarrow f''(x)$  is negative on  $(a,b)$   
 $f'(x)$  is increasing  $\Leftrightarrow f''(x)$  is positive on  $(a,b)$

Ex: Find the intervals of concavity:

$$f(x) = 2x^6 - 5x^4 + 7x - 3$$

$$\begin{aligned}
 f'(x) &= 12x^5 - 20x^3 + 7 \\
 f''(x) &= 60x^4 - 60x^2 = 0 \\
 60x^2(x^2 - 1) &= 0
 \end{aligned}$$

$$60x^2(x-1)(x+1) = 0$$

$$\begin{aligned} x^2 &= 0 & x &= \pm 1 \\ x &= 0 \end{aligned}$$



$f''$	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$60x^2$	+	-2	$-\frac{1}{2}$	$\frac{1}{2}$
$x-1$	-	-	-	+
$x+1$	-	+	+	+
$f''$	+	-	-	+

Below the table, there are four blue U-shaped arrows indicating the sign changes of  $f''$  across the intervals. A green arrow points from the text "inflection pts." to the second row of the table.

Def:  $x=c$  is an inflection point of  $f(x)$  if  $c$  is in the domain of  $f$  and  $f''(x)$  changes its sign at  $c$ . [point  $(c, f(c))$ ]

Ex: Find inflection pts:

$$\bullet f(x) = 3x^5 - 5x^4 - 1$$

$$f'(x) = 15x^4 - 20x^3$$

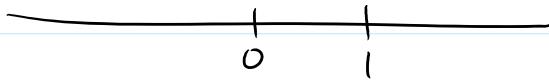
$$f''(x) = 60x^3 - 60x^2 = 0$$

$$60x^2(x-1) = 0$$

$$x = 0, 1$$



$$x=0, 1$$



$f''$	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
$60x^2$	+	-	+
$(x-1)$	-	-	+
$f''$	-	-	+

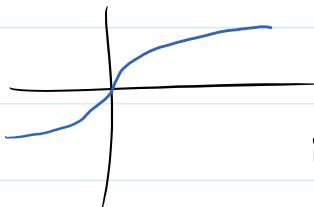
Below the table, there are three blue curved arrows: one pointing down from the first row between 0 and 1, another pointing down from the second row between 0 and 1, and a third pointing up from the third row after 1.

$x=1$  is an inflection pt

$$f(1) = 3 \cdot 1^5 - 5 \cdot 1^4 - 1 = -3$$

$\boxed{(1, -3)}$

$$\begin{aligned} g(x) &= \sqrt[3]{x} \\ &= x^{\frac{1}{3}} \end{aligned}$$



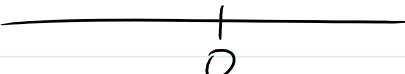
Dom:  $(-\infty, \infty)$

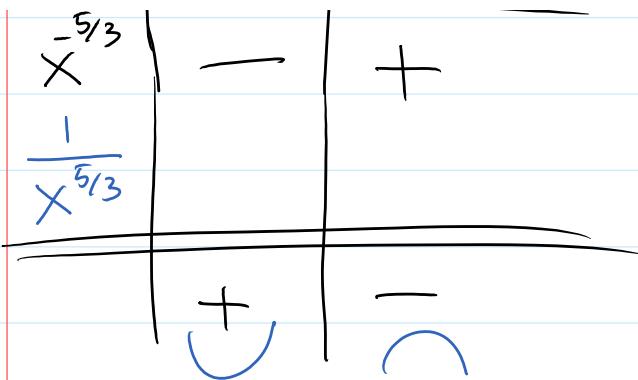
$$g'(x) = \frac{1}{3} x^{-\frac{2}{3}}$$

$$g''(x) = \frac{1}{3} \cdot \left(-\frac{2}{3}\right) x^{-\frac{5}{3}} = \frac{-2}{9} x^{-\frac{5}{3}} = \frac{-2}{9 x^{\frac{5}{3}}} = \frac{-2}{9 x \cdot \sqrt[3]{x^2}}$$

$x=0$  } do not mind  
the exponent

$g''$	$(-\infty, 0)$	$(0, \infty)$
$-\frac{2}{9}$	-	-
$x^{-\frac{5}{3}}$	-	+





$$\boxed{x=0} \text{ - inflection pt.}$$

$$f(0) = \sqrt[3]{0} = 0$$

$$\boxed{(0, 0)}$$

Ex:  $f(x) = 3x^4 - 2x^3 - 12x^2 + 18x + 15,$

Find intervals inc/dec, concave up/down;  
relative extrema, inflection points.

Dom:  $(-\infty, \infty)$

$$f'(x) = 12x^3 - 6x^2 - 24x + 18$$

$$f''(x) = 36x^2 - 12x - 24$$

$$f'(x) = 0$$

$$12x^3 - 6x^2 - 24x + 18 = 0$$

$$6(2x^3 - x^2 - 4x + 3) = 0$$

"rational zeros thm"

$x=1$  is a root, let's do  
synt division:

$$6(x-1)(2x^2+x-3) = 0$$

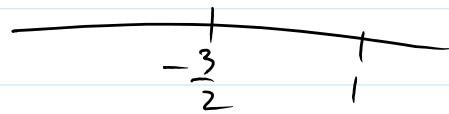
$$6(x-1)(2x+3)(x-1) = 0$$

$$\begin{array}{r|rrrr} & 2 & -1 & -4 & 3 \\ 1 & \downarrow & 2 & 1 & -3 \\ \hline 2 & 1 & -3 & 0 \end{array}$$

$$x=1, -\frac{3}{2}, 1$$

$$x=1, -\frac{3}{2}, 1$$

$\leftarrow \uparrow \downarrow \leftarrow \downarrow$



$f'$	$(-\infty, -\frac{3}{2})$	$(-\frac{3}{2}, 1)$	$(1, \infty)$
$6(2x+3)$	-	-2	+
$(x-1)^2$	+	+	+

$x = -\frac{3}{2} \Leftrightarrow \text{rel. min}$

$$f(-\frac{3}{2}) = -17.06$$

$$\boxed{(-\frac{3}{2}, -17.06)}$$

Concavity:

$$f''(x) = 36x^2 - 12x - 24 = 0$$

$$12(3x^2 - x - 2) = 0$$

$$12(3x+2)(x-1) = 0$$

$$x = -\frac{2}{3}, 1$$



$f''$	$(-\infty, -\frac{2}{3})$	$(-\frac{2}{3}, 1)$	$(1, \infty)$
$12(3x+2)$	-	-2	+
$x-1$	-	-	+
	+	-	+

$x = -\frac{2}{3}$  } Inflection points  
 $x = 1$  }

$$f(-\frac{2}{3}) = -1.15$$

$$f(1) = 22$$

$$\text{Inflection pts: } \boxed{(-\frac{2}{3}, -1.15), (1, 22)}$$

Inflection pts:  $\boxed{(-\frac{2}{3}, -1.15), (1, 22)}$