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Wednesday, October 11, 2017 12:01 PM

Section 3.4

Ex: Find the abs min/max of $f(x) = 2x^3 + 3x^2 - 12x - 7$ on the interval $-3 \le x \le 0$.

(1) Find critical points:

$$f'(x) = 0$$
$$6x^2 + 6x - 12 = 0$$
$$6(x^2 + x - 2) = 0$$
$$6(x-1)(x+2) = 0$$
$$x = 1, -2$$

Pick the critical num. in [-3,0]:

$$\boxed{x = -2}$$

(2) To find the abs min/max evaluate $f(x)$ at • and the endpts.

x	$f(x) = 2x^3 + 3x^2 - 12x - 7$
-2	$2\cdot(-8) + 3\cdot 4 + 24 - 7 = -16 + 12 + 24 - 7 = 13$ ← abs. max
-3	$2\cdot(-27) + 3\cdot 9 + 36 - 7 = -54 + 27 + 36 - 7 = 2$

-3	$2\cdot(-27)+3\cdot 9+36-7=-54+27+36-7$ $=2$
0	-7 ← abs. min

Ex: Find abs. max/min of

$$f(x)=x^2+\frac{16}{x} \quad \text{on} \quad (0,\infty).$$

Since $f(0)$ DNE and $f(\infty)$ DNE, let's find the limits:

$$\lim_{x\to 0^+} x^2+\frac{16}{x}=(0^+)^2+\frac{16}{0^+}=\frac{16}{0^+}=+\infty$$

$$\lim_{x\to\infty} x^2+\cancel{\frac{16}{x}}^{\to 0}=(\infty)^2+0=\infty$$

No abs max since the limits are $+\infty$, let's find the abs. min: (rel. min)

$$f(x)=x^2+\frac{16}{x}=x^2+16x^{-1}$$
$$f'(x)=2x-16x^{-2}=2x-\frac{16}{x^2}$$

$$2x-\frac{16}{x^2}=0$$

$$2x^3 \quad 16$$

$$\frac{2x^3}{x^2} - \frac{16}{x^2} = 0$$

$$\frac{2x^3 - 16}{x^2} = 0$$

$2x^3 - 16 = 0$

$x^3 = 8$

$x = 2$

$x^2 = 0$

$x = 0$

Critical numbers: $x = 2$.

($x = 0$ is not a crit. number since 0 is not in the domain)

$x = 2$ is in $(0, \infty)$

x	$f(x)$
2	$2^2 + \frac{16}{2} = 4 + 8 = \boxed{12}$
0^+	∞
∞	∞

abs. min

Def: (Elasticity of demand)

If $q = D(p)$ units of a comodity are demanded by the market at a unit price p where D is a differentiable function demanded by the market at a unit price p, where D is a differentiable function, then the price elasticity of demand for the commodity is given by

$$E(p) = -\frac{p}{q}\frac{dq}{dp} = -\frac{p \cdot q'}{q}$$

and has the interpretation

$$E(p) \approx \left[\begin{array}{l}\text{percentage rate of decrease in demand } q \\ \text{produced by a 1\% increase in price } p\end{array}\right]$$

<u>Ex:</u> $q = 240 - 2p \quad (0 \le p \le 120)$

a) Find $E(p)$

b) Find and interpret $E(100)$, $E(50)$

a) $$E(p) = \frac{-p \cdot q'}{q} = -\frac{p \cdot (240-2p)'}{240-2p} = \frac{2p}{240-2p}$$

$$= \boxed{\frac{p}{120-p}}$$

b) $p = 100$: $E(100) = \dfrac{100}{20} = 5$

If the price is 100 and we increase

If the price is 100 and we increase it by 1%, the demand decreases by 5%.

$p=50: E(50) = \frac{50}{70} = \frac{5}{7} \approx 0.71$

If the price is 50 and we inc. it by 1%, the demand decreases by 0.71%.

Levels of Elasticity:

$E(p) > 1$: Elastic demand. The demand is relatively sensitive to changes in price. Revenue ↘ as price ↗

$E(p) < 1$: Inelastic demand. The demand is relatively insensitive to changes in price.
Revenue ↗ as price ↗

$E(p) = 1$: Unitary demand. The relative changes in price and demand are (approximately) equal Revenue ⟶ as price ↗