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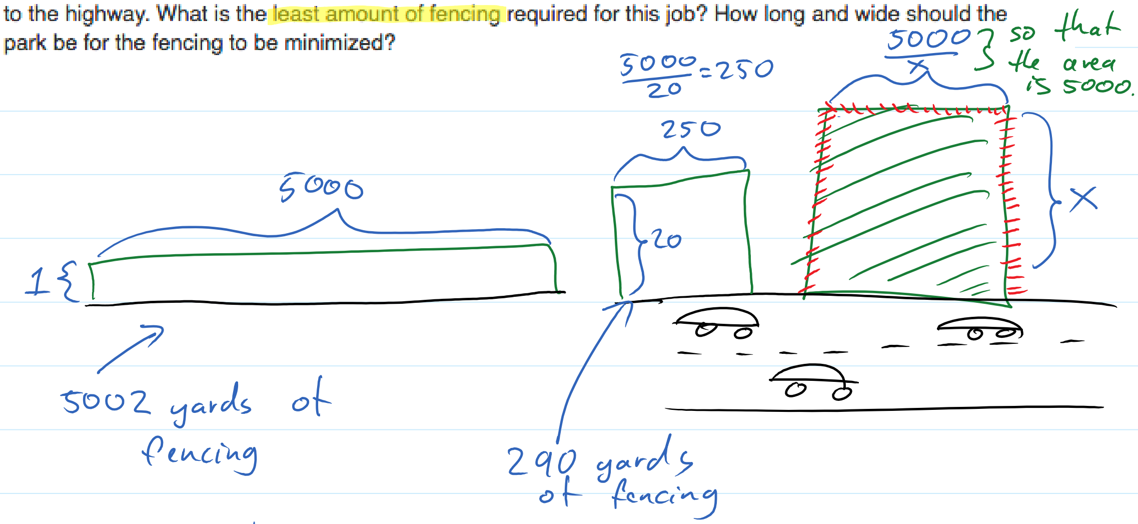
Friday, October 13, 2017 11:58 AM

3.5 Optimization

1) Undersand the problem → write it in math
2) Find a function of <u>one</u> variable, that you can optimize
3) Optimize the function

EXAMPLE 3.5.1 Minimizing Amount of Fence

The highway department is planning to build a picnic park for motorists along a major highway. The park is to be rectangular with an area of 5,000 square yards and is to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing required for this job? How long and wide should the park be for the fencing to be minimized?



total amount of fencing $f(x) = x + \frac{5000}{x} + x = 2x + 5000x^{-1}$

$$f'(x) = 2 - 5000x^{-2} = 2 - \frac{5000}{x^2} = 0$$

$$\frac{2x^2 - 5000}{x^2} = 0$$

$$2x^2 - 5000 = 0$$
$$x^2 = 2500$$
$$x = \pm 50$$

→ Since x is length,

$x > 0$

$\boxed{x=50}$ is it minimum?

2nd der. test: $f''(x) = 5000 \cdot 2x^{-3} = \dfrac{10000}{x^3}$

$f''(50) > 0 \Rightarrow x=50$ is a minimum.

width: $\boxed{50}$

Length: $\dfrac{500\cancel{0}}{5\cancel{0}} = \boxed{100}$

Fencing needed: $f(50) = 2 \cdot 50 + \dfrac{5000}{50} = 100+100$

$= \boxed{200}$

EXAMPLE 3.5.2 Maximizing Profit

Mateo owns a small company that makes souvenir T-shirts. He can produce the shirts at a cost of \$2 apiece. The shirts have been selling for \$5 apiece, and at this price, tourists have been buying 4,000 shirts a month. Mateo plans to raise the price of shirts and expects that for each \$1 increase in price, 400 fewer shirts will be sold each month. What price should Mateo charge per shirt to maximize profit?

price	quantity $= \boxed{q}$	profit
5	4000	$5 \cdot 4000 - 2 \cdot 4000$ $(5-2)\,4000$
6	$4000-400$	$6 \cdot q - 2 \cdot q = (6-2)\,q$
7	$4000 - 2 \cdot 400$	$(7-2)\,q$
x	$4000-(x-5)400$	$(x-2)\left(4000-(x-5)400\right)$

$x \quad \Big| \; 4000-(x-5)400 \; \Big| \quad (x-2)\big(4000-(x-5)400\big)$

Profit

$$P(x) = (x-2)\big(4000-(x-5)400\big)$$

$$= (x-2)(4000 - 400x + 2000)$$
$$= (x-2)(6000 - 400x)$$
$$= -400x^2 + 6800x - 12000$$

$$P'(x) = -800x + 6800 = 0$$

$$x = \frac{-6800}{-800} = \frac{68}{8} = \frac{34}{4} = \frac{17}{2}$$

$$x = \boxed{8.5}$$

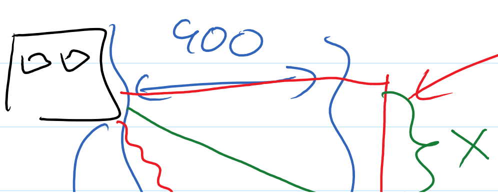
$$P''(x) = -800$$ → 2^{nd} der test $P''(8.5) < 0$

⇒ $x = 8.5$ is maximum.

$$\boxed{\$8.50}$$

EXAMPLE 3.5.5 Minimizing Cost of Construction

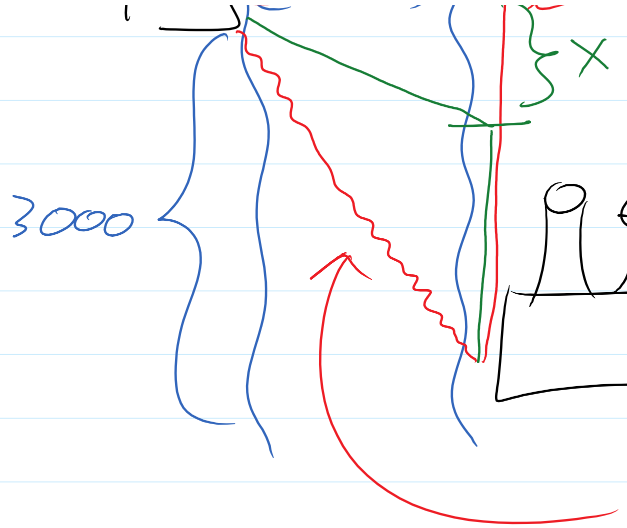
A cable is to be run from a power plant on one side of a river 900 meters wide to a factory on the other side, 3,000 meters downstream. The cost of running the cable under the water is \$5 per meter, while the cost over land is \$4 per meter. What is the most economical route over which to run the cable?



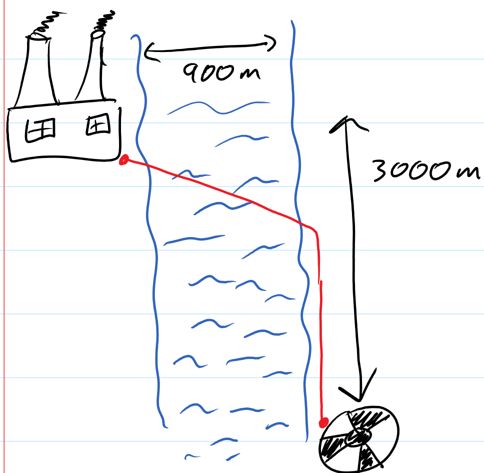
cost: $\underbrace{3000 \cdot 4}_{\text{Land}} + \underbrace{900 \cdot 5}_{\text{water}}$

land water

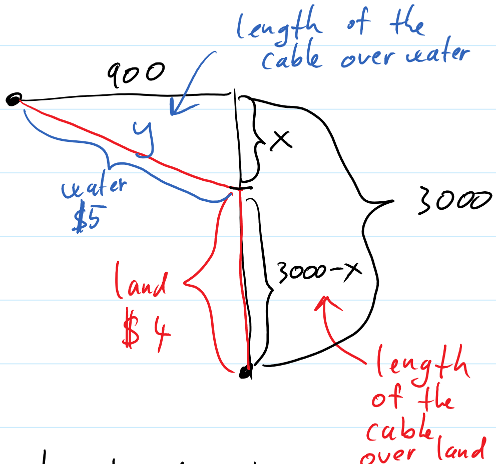
$= 16500$



cost : $\underbrace{5 \cdot 3132.09}_{\text{water}} = 15660$



$\Longrightarrow$



$$\text{Total cost} = \underbrace{\text{land cost}}_{4(3000-x)} + \underbrace{\text{water cost}}_{5\cdot y}$$

We need to minimize $T(x) = 4x + 5y$.

To have a function of one variabe, let's use the Pythagorean thm:

$$y^2 = 900^2 + x^2$$

$$y = \sqrt{900^2 + x^2}$$

$$T(x) = 4x + 5\left(900^2 + x^2\right)^{1/2}$$

$$T'(x) = 4 + 5 \cdot \frac{1}{\cancel{2}}\left(900^2 + x^2\right)^{-1/2} \cdot \cancel{2}x$$

$$= 4 + \frac{5x}{\sqrt{900^2 + x^2}}$$

let's find minimum:

$$T'(x) = 0$$

$$\frac{4\sqrt{900^2 + x^2} + 5x}{\sqrt{900^2 + x^2}} = 0$$

$$4\sqrt{900^2 + x^2} + 5x = 0$$

$$\left(\sqrt{900^2 + x^2}\right)^2 = \left(\frac{5}{4}x\right)^2$$

$$900^2 + x^2 = \frac{25}{16}x^2$$

$$900^2 = \frac{9}{16}x^2$$

$$x^2 = 900^2 \cdot \frac{16}{9}$$

$$x^2 = 1440000$$

$$x = \pm 1200$$

Since x is length, then $x = 1200$. The length of the cable over land is $3000 - 1200 = \boxed{1800\,m}$

The length of the cable over water is $\sqrt{900^2+x^2}$

$$= \sqrt{900^2+1200^2}$$

$$= \boxed{1500 \text{ m}}$$