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Wednesday, October 18, 2017 11:53 AM

Section 4.3

Def:

$$\begin{cases} \dfrac{d}{dx}(e^x) = e^x \\ (e^x)' = e^x \end{cases}$$

$$\begin{cases} \dfrac{d}{dx}\left(e^{u(x)}\right) = e^{u(x)} \cdot \dfrac{d}{dx} u(x) \\ \left(e^{u(x)}\right)' = e^{u(x)} \cdot u'(x) \end{cases}$$

Ex: Differenciate:

- $f(x) = x^2 e^x$

$$f'(x) = e^x (x^2)' + x^2 (e^x)'$$
$$= \boxed{e^x \cdot 2x + x^2 e^x} = \boxed{x e^x (2 + x)}$$

Crit. pts: $x = 0, -2$

- $g(x) = \dfrac{x^3}{e^x + 2}$

$$(e^x+2)(x^3)' - x^3(e^x+2)' \qquad (e^x+2)3x^2 - x^3 e^x$$

$$g'(x) = \frac{(e^x+2)(x^3)' - x^3(e^x+2)'}{(e^x+2)^2} = \frac{(e^x+2)3x^2 - x^3e^x}{(e^x+2)^2}$$

$$= \frac{x^2(3e^x+6-xe^x)}{(e^x+2)^2}$$

$$\boxed{(e^{u(x)})' = e^{u(x)} \cdot u'(x)}$$

- $h(x) = e^{x^2+1}$

$$h'(x) = e^{x^2+1} \cdot (x^2+1)' = e^{x^2+1} \cdot 2x$$

$$= \boxed{2x\, e^{x^2+1}}$$

$$\boxed{e^{\ln x} = x}$$

- $f(x) = 2^x = (e^{\ln 2})^x = e^{x\ln 2}$

$$f'(x) = e^{x\ln 2} \cdot (x\underbrace{\ln 2}_{\text{constant}})' = e^{x\ln 2} \cdot \ln 2 (x)'$$

$$= e^{x\ln 2} \cdot \ln 2 \cdot 1 = \boxed{2^x \cdot \ln 2}$$

Def:

$$\frac{d}{dx}(b^x) = b^x \cdot \ln b, \text{ where } b>0 \text{ and } b\neq 1.$$

Look at the derivative of $f(x) = x^x$.

Look at the derivative of $f(x)=x^x$.

Ex: Find abs min/max of xe^{2x} on $[-1,1]$.

$$(xe^{2x})' = e^{2x}\cdot(x)' + x(e^{2x})' = e^{2x} + xe^{2x}\cdot\underbrace{2}_{(2x)'}$$

$$= e^{2x}(1+2x) = 0$$

$$e^{2x}=0$$
$$\ln e^{2x} = \ln 0$$
$$2x \ln e = \ln 0$$
$$x = \frac{\ln 0}{2}$$
No sol

$$1+2x=0$$
$$x = -\frac{1}{2}$$

crit pts in $[-1,1]$: $x=-\frac{1}{2}$

x	$f(x)=xe^{2x}$
-1	$-e^{-2} = \frac{-1}{e^2} \simeq -0.135$
1	$e^2 \simeq 7.389$
$-\frac{1}{2}$	$-\frac{1}{2}e^{-2\cdot(\frac{1}{2})} = -\frac{1}{2}e^{-1} \simeq -0.184$

max → 1 | e^2

min → $-\frac{1}{2}$

Def: $\begin{cases} \frac{d}{dx}(\ln x) = \frac{1}{x} \\ (\ln x)' = \frac{1}{x} \end{cases}$

Find $x = e^{\ln x}$ $\Big\downarrow$ diff. both sides:

$$1 = \left(e^{\ln x}\right)'$$
$$1 = e^{\ln x} \cdot (\ln x)'$$

$$1 = x \cdot (\ln x)'$$
$$\frac{1}{x} \quad \frac{}{x}$$

$$\boxed{(\ln x)' = \frac{1}{x}}$$

Exs Differenciate:

- $f(x) = x \cdot \ln x$

$$f'(x) = \ln x (x)' + x \cdot (\ln x)'$$
$$= \ln x + x \cdot \frac{1}{x} = \boxed{1 + \ln x}$$

- $g(x) = (x + \ln x)^{3/2}$

$$\boxed{(a+b)^2 \neq a^2 + b^2}$$

$$g'(x) = \frac{3}{2}(x + \ln x)^{\frac{1}{2}} \cdot (x + \ln x)'$$
$$= \boxed{\frac{3}{2}(x + \ln x)^{1/2} \cdot \left(1 + \frac{1}{x}\right)}$$

$$= \boxed{\frac{3}{2}(x+\ln x)^{1/2}\cdot\left(1+\frac{1}{x}\right)}$$

$$\cdot f(x) = \frac{\ln\sqrt[3]{x^2}}{x^4} = \frac{\ln x^{2/3}}{x^4} = \frac{\frac{2}{3}\ln x}{x^4}$$

$$= \frac{2}{3}\cdot\frac{\ln x}{x^4} = \frac{2}{3}\cdot x^{-4}\ln x$$

$$f'(x) = \frac{2}{3}\cdot\left(\frac{\ln x}{x^4}\right)' = \frac{2}{3}\cdot\frac{x^4\cdot\frac{1}{x} - (\ln x)\cdot 4x^3}{x^8}$$

$$= \frac{2}{3}\,\frac{x^3 - 4x^3\ln x}{x^8} = \frac{2}{3}\,\frac{x^{\cancel{3}}(1-4\ln x)}{x^{\cancel{8}\,5}}$$

$$= \boxed{\frac{2}{3}\cdot\frac{1-4\ln x}{x^5}} = \boxed{\frac{2}{3}\cdot x^{-5}\cdot(1-4\ln x)}$$