Section 4.3

Differenciate:

$$f(x) = \ln(2x^3+1)$$

$$f'(x) = \frac{1}{2x^3+1} \cdot (2x^3+1)'$$

Def: $$\begin{cases} \frac{d}{dx}\left(\ln(u(x))\right) = \frac{1}{u(x)} \cdot \frac{d}{dx} u(x) \\ (\ln(u(x))' = \frac{u'(x)}{u(x)} \end{cases}$$

How to diff. 2^x, $\log_3 x$?

$$2 = e^{\ln 2}$$

$$2^x = \left(e^{\ln 2}\right)^x = e^{(\ln 2)\cdot x}$$

$$3 \cdot 9 = 3 \cdot 3^2 = 3^3$$

$$e^{\ln x} = x$$

$$(2^x)' = \left(e^{(\ln 2)\cdot x}\right)'$$

$$= e^{(\ln 2)\cdot x} \cdot \left[(\ln 2)\cdot x\right]' = e^{(\ln 2)\cdot x} \cdot \ln 2 \cdot (x)'$$

$$= e^{(\ln 2)\cdot x} \cdot \left[(\ln 2)\cdot x\right]' = e^{(\ln 2)x} \cdot \ln 2 \cdot (x)'$$

$$= e^{(\ln 2)x} \cdot \ln 2 = \boxed{2^x \cdot \ln 2}$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\log_2 x = \frac{\ln x}{\ln 2}$$

$$(\log_2 x)' = \left(\frac{\ln x}{\ln 2}\right)' = \frac{1}{\ln 2} \cdot (\ln x)' = \frac{1}{\ln 2} \cdot \frac{1}{x}$$

$$= \boxed{\frac{1}{x \cdot \ln 2}}$$

$$\begin{cases} (b^x)' = b^x \cdot \ln b \\ (\log_b x)' = \dfrac{1}{x \cdot \ln b} \end{cases}$$

Differenciate:

- $f(x) = 5^{2x-3}$

$$f'(x) = 5^{2x-3} \cdot \ln 5 \cdot (2x-3)' = 5^{2x-3} \cdot \ln 5 \cdot 2$$

$$2(\ln 5) \, 5^{2x-3}$$

$$= 2(\ln 5)\, 5^{2x-3}$$

- $g(x) = (x^2 + \log_7 x)^4$

$$g'(x) = 4(x^2 + \log_7 x)^3 \cdot (x^2 + \log_7 x)'$$

$$= 4(x^2 + \log_7 x)^3 \cdot \left(2x + \frac{1}{x \cdot \ln 7}\right)$$

Algebra with log..

$$\log(A \cdot B) = \log A + \log B$$

$$\log \frac{x}{y} = \log x - \log y$$

logarithmic differentiation

$$y(x) = y = x^2 (x-3)^3$$

How could we find y' without the prod. rule?

$$y = x^2 (x-3)^3 \qquad // \ln$$

$$y = x^2 (x-3)^3 \qquad // \ln$$

$$\ln y = \ln\left(x^2 (x-3)^3\right)$$

$$\ln y = \ln x^2 + \ln (x-3)^3$$

$$\ln y = 2\ln x + 3\ln(x-3)$$

differenciate

$$\frac{y'(x)}{y(x)} = \frac{2}{x} + \frac{3}{x-3}\cdot(x-3)'$$

$$\boxed{\left(\ln(y(x))\right)' = \frac{1}{y(x)}\cdot(y(x))' = \frac{y'(x)}{y(x)}}$$

$$\frac{y'(x)}{y(x)} = \frac{2}{x} + \frac{3}{x-3}$$

solve for $y'(x)$

$$y' = y(x)\left(\frac{2}{x} + \frac{3}{x-3}\right)$$

$y(x) = y = x^2(x-3)^3$

$$y' = x^2(x-3)^3\left(\frac{2}{x} + \frac{3}{x-3}\right)$$

Differenciate:

$$y = \frac{x^2 (2x-1)^5}{(x^2+1)^3 (3x+1)^4}$$

$$y = \frac{\qquad}{(x^2+1)^3\,(3x+1)^4}$$

$$\ln y = \ln\left(\frac{x^2(2x-1)^5}{(x^2+1)^3(3x+1)^4}\right)$$

$$\ln y = \ln\left(x^2(2x-1)^5\right) - \ln\left((x^2+1)^3(3x+1)^4\right)$$

$$\ln y = \ln x^2 + \ln(2x-1)^5 - \left[\ln(x^2+1)^3 + \ln(3x+1)^4\right]$$

$$\ln y = 2\ln x + 5\ln(2x-1) - 3\ln(x^2+1) - 4\ln(3x+1)$$

$$\frac{y'}{y(x)} = \frac{2}{x} + \frac{5}{2x-1}\cdot 2 - \frac{3}{x^2+1}(2x) - \frac{4}{3x+1}\cdot(3)$$

$$y' = y(x)\left[\frac{2}{x} + \frac{10}{2x-1} - \frac{6x}{x^2+1} - \frac{12}{3x+1}\right]$$

$$y' = \boxed{\frac{x^2(2x-1)^5}{(x^2+1)^3(3x+1)^4}\left[\frac{2}{x} + \frac{10}{2x-1} - \frac{6x}{x^2+1} - \frac{12}{3x+1}\right]}$$